Solucionario Trigonometría 1.°

APLICAMOS LO APRENDIDO (página 5) Unidad 1

1. Colocando los ángulos en el sentido antihorario:

$-(-7x) + 89° = 180°$
$7x = 180° - 89°$
$7x = 91°$
$\Rightarrow x = 13°$

Clave C

2. Colocando los ángulos en el sentido antihorario:

$-(-49°) + (2x + 1°) = 90°$
$49° + 2x + 1° = 90°$
$2x = 40°$
$\Rightarrow x = 20°$

Piden:
$\dfrac{x + 1°}{3} = \dfrac{20° + 1°}{3} = \dfrac{21°}{3} = 7°$

Clave B

3. Colocando los ángulos en el sentido antihorario:

$x + -(-19°) + -(-5°) = 40°$
$x + 19° + 5° = 40°$
$x + 24° = 40°$
$\Rightarrow x = 16°$

Clave D

4. Colocando los ángulos en el sentido antihorario:

$3x + -(-5x) + 4x = 180°$
$3x + 5x + 4x = 180°$
$12x = 180°$
$\Rightarrow x = 15°$

Clave A

5. Colocando los ángulos en el sentido antihorario:

$-(-21°) + -(-39°) + 3\alpha = 90°$
$21° + 39° + 3\alpha = 90°$
$60° + 3\alpha = 90°$
$3\alpha = 30°$
$\Rightarrow \alpha = 10°$

Piden: $\alpha + 1° = 10° + 1° = 11°$

Clave E

6. Colocando los ángulos en el sentido antihorario:

$3\theta + -(2\alpha) = 5x$
$3\theta - 2\alpha = 5x$
$\Rightarrow x = \dfrac{3\theta - 2\alpha}{5}$

Clave D

7. Los ángulos tienen el mismo sentido (antihorario):

$2x + 7° = 3x - 8°$
$\Rightarrow 15° = x$

Clave C

8. Colocando los ángulos en el sentido antihorario:

$140° + -(2° - 3x) + -(-150°) = 360°$
$140° - (2° - 3x) - (-150°) = 360°$
$140° - 2° + 3x + 150° = 360°$
$3x = 72°$
$\Rightarrow x = 24°$

Clave D

9. Colocando los ángulos en el sentido antihorario:

$-(-73°) + 90° + (3x + 2°) = 180°$
$73° + 90° + 3x + 2° = 180°$
$3x = 15°$
$\Rightarrow x = 5°$

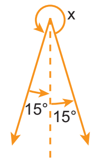
Clave A

10. Como $\overleftrightarrow{OS}$ es bisectriz: $m\angle TOS = m\angle SOR$, en el mismo sentido antihorario, se tendrá:

$38° - 5x = -(x - 30°)$
$38° - 5x = -x + 30°$
$8° = 4x$
$\Rightarrow x = 2°$

Clave B

11. Usar los datos y completar ángulos.
Se observa:

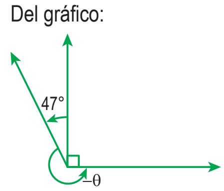


$x + 15° + 15° = 360°$
$x + 30° = 360°$
$x = 360° - 30°$
$\therefore\ x = 330°$

Clave E

12. Cambio en el sentido de giro de θ.

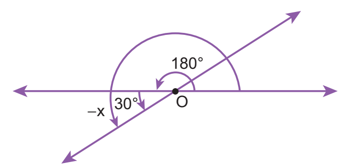
Del gráfico:



$90° + 47° + (-\theta) = 360°$
$90° + 47° - \theta = 360°$
$-\theta = 360° - 90° - 47°$
$-\theta = 223°$
$\therefore\ \theta = -223°$

Clave C

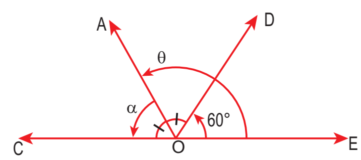
13. Completando ángulos y cambiando el sentido de x.
Del gráfico:



$180° + 30° = -x$
$\therefore\ x = -210°$

Clave A

14. En el gráfico:



$\theta + \alpha = 180°$ ... (1)

Además:
$60° + \alpha + \alpha = 180°$
$2\alpha = 180° - 60°$
$2\alpha = 120$
$\alpha = 60°$ ... (2)

(2) en (1):
$\theta + 60° = 180°$
$\therefore\ \theta = 120°$

Clave D

PRACTIQUEMOS
Nivel 1 (página 7) Unidad 1

Comunicación matemática

1. Los ángulos que giran en sentido horario siguen el sentido de giro de las manecillas del reloj.

Clave D

2. Por convención será positivo si los ángulos trigonométricos giran en sentido antihorario y negativo si lo hacen en sentido horario.

Clave C

Razonamiento y demostración

3. Colocando los ángulos en el sentido antihorario:

$-(-50°) + x = 90°$
$50° + x = 90°$
$\therefore\ x = 40°$

Clave A

4. Colocando los ángulos en el sentido antihorario:

$x + -(-x) + -(-x) = 90°$
$x + x + x = 90°$
$3x = 90°$
$\therefore\ x = 30°$

Clave A

5. Colocando los ángulos en el sentido antihorario:

$x - 10° = -(-20°)$
$x - 10° = 20°$
$\therefore\ x = 30°$

Clave E

6. Colocando los ángulos en el sentido antihorario:

$x + (-\alpha) = 90°$
$x - \alpha = 90°$
$\therefore\ x = 90° + \alpha$

Clave B

7. Colocando los ángulos en el sentido antihorario:

$x + 50° + -(10° - x) = 90°$
$x + 50° - 10° + x = 90°$
$2x = 50°$
$\therefore\ x = 25°$

Clave A

Resolución de problemas

8. Del gráfico: $\alpha + \theta = 90°$, además:
$\alpha = 3\theta$, se tiene:
$\alpha + \theta = 3\theta + \theta = 90°$

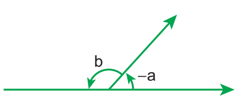
$4\theta = 90°$

$\therefore\ \theta = \dfrac{45°}{2}$

Piden valor de $-\theta$; $-\theta = -\dfrac{45°}{2}$

Clave C

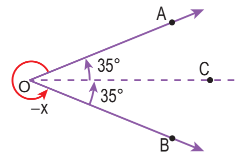
9. Por dato: $a + b = 20°$ ... (1)
Cambio de sentido del ángulo a.



Del gráfico: $b - a = 180°$ ... (2)
De (1) y (2):
$(a + b) + (b - a) = 200°$
$2b = 200°$
$b = 100° \ \wedge\ a = -80°$
Finalmente: $3a = -240°$

Clave D

10. Completando con los datos y cambio de orientación de giro.



En el gráfico:
$2(35°) + (-x) = 360°$
$x = -290°$
$\therefore\ 20° - x = 310°$

Clave A

Nivel 2 (página 7) Unidad 1

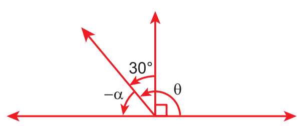
Comunicación matemática

11. En la figura α y θ son ángulos en sentido horario (negativos), entonces:

$\alpha < 0°$
$\theta < 0°$ $\Rightarrow$ $\alpha + \theta < 0°$
$\beta < 0°$

Se concluye que β también es negativo y por lo tanto su sentido de giro es horario.

12. Cambiemos el sentido de giro de α.



De la figura:
$30° + (-\alpha) = 90°$
$\alpha = -60°$
También:
$-\alpha + \theta = 180°$
$\theta = 180° + \alpha$
$\theta = 180° - 60°$
$\theta = 120°$

Luego:
- θ es un ángulo positivo $\Rightarrow$ I (F)
- $-\theta + \alpha = -120° + (-60°) = -180° \Rightarrow$ II (V)
- $\alpha + \theta = -60° + (120°) = 60° \Rightarrow$ III (V)

Clave B

Razonamiento y demostración

13. Colocando los ángulos en el sentido antihorario:

$30° + -(-20°) = x$
$30° + 20° = x$
$\Rightarrow 50° = x$

Clave E

14. Colocando los ángulos en el sentido antihorario:

$2x = -\theta + \alpha$
$\therefore\ x = \dfrac{\alpha - \theta}{2}$

Clave B

15. Colocando los ángulos en el sentido antihorario:

$(3x + 30°) + 90° + -(30° - 6x) = 180°$
$3x + 30° - 30° + 6x = 90°$
$9x = 90°$
$\therefore\ x = 10°$

Clave E

16. Colocando los ángulos en el sentido antihorario:

$\alpha + 90° + (-\theta) = 180°$
$\therefore\ \alpha - \theta = 90°$

Clave D

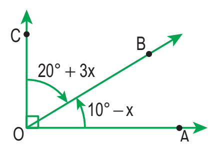
17. Como $\overleftrightarrow{OT}$ es bisectriz: $m\angle BOT = m\angle TOA$, en el mismo sentido antihorario, se tendrá:

$6x - 8° = -(4x - 12°)$
$6x - 8° = -4x + 12°$
$10x = 20°$
$\therefore\ x = 2°$

Clave B

Resolución de problemas

18. De los datos:



Si $\angle AOB$ tiene sentido antihorario entonces:

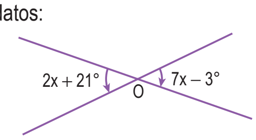
$10° - x - \underbrace{(20° + 3x)}_{\text{Giro horario}} = 90°$

$10° - x - 20° - 3x = 90°$
$-10° - 4x = 90°$
$4x = -10° - 90°$
$x = \dfrac{-100°}{4}$
$\therefore\ x = -25°$

Observación:
Si $\angle AOB$ tiene sentido horario, el valor de x sale positivo.

Clave A

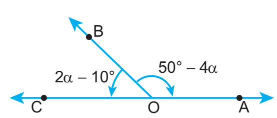
19. De los datos:



Tienen sentidos de giro opuestos, entonces:
$2x + 21° = -(7x - 3°)$
$2x + 21° = -7x + 3°$
$9x = -18°$
$x = -2°$
Piden:
$\therefore\ 3x + 2 = 3(-2°) + 2° = -4°$

Clave E

20. De los datos:



De la figura:

$$2\alpha - 10° + \underbrace{(4\alpha - 50°)}_{\text{Cambio de sentido}} = 180°$$

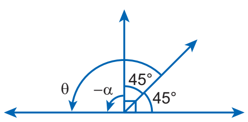
$$2\alpha - 10° + 4\alpha - 50° = 180°$$
$$6\alpha = 180° + 60°$$
$$\alpha = \frac{240°}{6}$$
$$\therefore\ \alpha = 40°$$

Clave E

Nivel 3 (página 8) Unidad 1

Comunicación matemática

21. Cambiando el sentido de α y completando la figura:



Luego:
$-\alpha = 90°$
$\alpha = -90°$ ... (1)

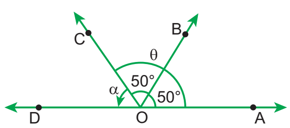
Además:
$\theta = 45° - \alpha$ ... (2)
$\theta = 45° - (-90°)$
$\theta = 135°$ ... (3)

De (1) y (3):
$\theta + \alpha = 135° + (-90°)$
$\theta + \alpha = 45°$ ... (4)

- De (1), $\alpha = -90°$
  $\therefore$ α es un ángulo recto negativo $\Rightarrow$ I (F)
- De (2), $\theta = 45° - \alpha \Rightarrow \theta + \alpha = 45°$
  $\therefore$ $\theta + \alpha$, tiene un sentido de giro antihorario. $\Rightarrow$ II (F)
- De (2): $\theta = 45° - \alpha \Rightarrow \alpha = 45° - \theta \Rightarrow$ III (V)

Clave E

22. Completando el gráfico:



Se observa:
$50° + 50° + \alpha = 180°$
$\alpha = 180° - 100°$
$\alpha = 80°$

Luego:
$\beta = \alpha + \theta = 80° + \theta$

Si β tiene sentido horario, entonces:
$\beta < 0°$
$\alpha + \theta < 0°$
$80° + \theta < 0°$
$\theta < -80°$ ... (1)

Si θ gira en sentido antihorario, $\theta = 100°$
Pero de (1): θ no cumple la desigualdad.
$\therefore$ θ gira en sentido horario.

Razonamiento y demostración

23. Colocando los ángulos en el sentido antihorario:

$10° + -(-12°) + 8° = \alpha$
$10° + 12° + 8° = \alpha$
$\therefore\ 30° = \alpha$

Clave A

24. Colocando los ángulos en el sentido antihorario:

$5x + 3x + -(-7x) = 360°$
$5x + 3x + 7x = 360°$
$15x = 360°$
$\therefore\ x = 24°$

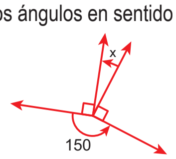
Clave A

25. Colocando los ángulos en el sentido antihorario:

$(x + 5°) + -(15° - x) + (20° + 3x) = 180°$
$x + 5° - 15° + x + 20° + 3x = 180°$
$5x = 170°$
$\therefore\ x = 34°$

Clave B

26. Cambiando los ángulos en sentido antihorario.



Luego: $x + 90° + 150° + 90° = 360°$
$x = 360° - 180° - 150°$
$\therefore\ x = 30°$

Clave B

27. Colocando los ángulos en el sentido antihorario:

$\alpha + -(-\beta) + \theta = x$
$\Rightarrow \alpha + \beta + \theta = x$

Clave D

28. $\beta - x - \alpha = 90°$
$-x = 90° + \alpha - \beta$
$x = \beta - \alpha - 90°$

Clave A

29. Del gráfico:

$(4n + 12)° - (2 - 7n)° = 120°$
$4n + 12 - 2 + 7n = 120$
$11n = 110$
$n = 10$

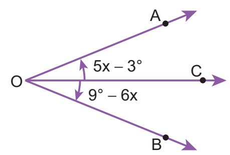
Clave B

30. Dato: $\theta + x = 60°$ ... (1)
Del gráfico: $\theta - x = 90°$ ... (2)
Sumando (1) y (2):
$2\theta = 150°$
$\theta = 75°$
$\Rightarrow x = -15°$

Clave D

Resolución de problemas

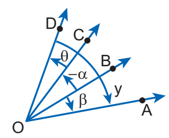
31. De los datos graficamos:



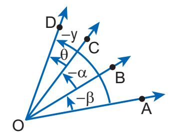
$\overleftrightarrow{OC}$ bisectriz, entonces, como los ángulos tienen sentidos opuestos:
$5x - 3° = -(9° - 6x)$
$5x - 3° = -9° + 6x$
$\therefore\ x = 6°$

Clave B

32. Graficando los datos:



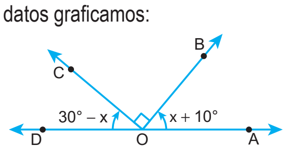
Cambiamos los ángulos a un mismo sentido de giro:



Luego:
$(-\beta) + (-\alpha) + \theta = -y$
$\therefore\ y = \alpha + \beta - \theta$

Clave E

33. De los datos graficamos:



Luego: cambiando el sentido de giro en $\angle COD$ se tiene que:

$$x + 10° + 90° + \underbrace{(x - 30°)}_{\text{Sentido antihorario}} = 180°$$

$2x = 110°$
$\therefore\ x = 55°$

Clave D

SISTEMAS DE MEDICIÓN ANGULAR

PRACTIQUEMOS

Nivel 1 (página 12) Unidad 1

Comunicación matemática

1.
I. El sistema sexagesimal está definido al dividir al ángulo de 1 vuelta en 360 partes iguales (F)
II. El número de radianes contenidos en una vuelta a 2π rad (F)
III. El sistema sexagesimal hace uso de subunidades para representar al ángulo, las cuales se definen:
1': minuto sexagesimal
1'': segundo sexagesimal
y están definidas:
$1' = 60''$ (F)

Clave B

2. De la definición de sistemas se obtiene la relación:

$m\angle 1\text{vuelta} = 360° = 400^g = 2\pi \text{ rad}$ ... (1)

Por dato:

θ es la tercera parte de una vuelta entonces:

$\theta = \frac{1}{3} m\angle 1\text{vuelta}$

En la relación (1):

$\frac{m\angle 1\text{vuelta}}{3} = 120° = \frac{400^g}{3} = \frac{2\pi}{3}\text{rad}$

$\theta = 120° = \frac{400^g}{3} = \frac{2\pi}{3}\text{rad}$

Por lo que se concluye:

$\theta = 120°$ sistema sexagesimal

$\theta = \frac{400^g}{3}$ sistema centesimal

$\theta = \frac{2\pi}{3}$ rad sistema radial

Clave C

Razonamiento y demostración

3. $\frac{\pi}{5}\text{rad}\left(\frac{180°}{\pi \text{ rad}}\right) = \frac{180°}{5} = 36°$

Clave E

4. $25^g \cdot \left(\frac{9°}{10^g}\right) = \frac{25.9°}{10} = 22,5°$

Clave D

5. $160^g \cdot \left(\frac{\pi \text{ rad}}{200^g}\right) = \frac{160\pi}{200}\text{ rad} = \frac{4\pi}{5}\text{ rad}$

Clave D

6. $54°\left(\frac{\pi \text{rad}}{180°}\right) = \frac{54\pi}{180}\text{ rad} = \frac{3\pi}{10}\text{ rad}$

Clave E

7. $81° \cdot \left(\frac{10^g}{9°}\right) = \frac{81.10^g}{9} = 90^g$

Clave E

8. $\frac{\pi}{8}\text{rad}\left(\frac{200^g}{\pi \text{ rad}}\right) = \frac{200^g}{8} = 25^g$

Clave D

9. $J = \frac{S+C}{R}$

Usando la relación entre S, C y R:

$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi} = k$

$\therefore\ J = \frac{380k}{\pi k} = \frac{380}{\pi}$

Clave D

10. $J = \frac{2C+3S}{C-S}$

Usando la relación entre S y C:

$J = \frac{2(10k)+3(9k)}{(10k)-(9k)} = \frac{20k+27k}{10k-9k}$

$\therefore\ J = \frac{47k}{k} = 47$

Clave D

11. $J = \frac{3C-S}{C-S}$

Usando la relación entre S y C:

$\frac{S}{9} = \frac{C}{10} = k$

$J = \frac{3(10k)-(9k)}{(10k)-(9k)} = \frac{30k-9k}{10k-9k} = \frac{21k}{k}$

$J = 21$

Clave C

Resolución de problemas

12. Por dato: $C = 130 \Rightarrow \frac{C}{200} = \frac{R}{\pi}$

$R = \frac{\pi C}{200} = \frac{130\pi}{200} = \frac{13\pi}{20}$

$\therefore$ La medida circular es $\frac{13\pi}{20}$ rad.

Clave A

13. Por dato: $C = 40$; sabemos que:

$\frac{S}{9} = \frac{C}{10}$

$\frac{S}{9} = \frac{40}{10} \Rightarrow S = 36$

$\therefore$ La medida sexagesimal es 36°.

Clave B

14. $S = 6x+3 \ \wedge\ C = 7x+2$

$\frac{S}{C} = \frac{6x+3}{7x+2} \Rightarrow \frac{9}{10} = \frac{6x+3}{7x+2} \Rightarrow x = 4$

Reemplazando en S:

$S = 6(4)+3$

$S = 27$

$\therefore$ El ángulo mide 27°.

Clave C

15. $S = nC \Rightarrow \frac{S}{C} = n \Rightarrow \frac{9}{10} = n$

$\Rightarrow n = 0,9$

Reemplazando en la expresión:

$E = 12(0,9) + 0,2 = 11$

Clave B

16. $7C - 4S = 34$; usando la relación entre S y C:

$7\left(\frac{10}{9}S\right) - 4S = 34 \Rightarrow \frac{70S}{9} - 4S = 34$

$\frac{34}{9}S = 34 \Rightarrow S = 9$

Entonces el ángulo mide 9°, su medida circular será:

$9° \cdot \left(\frac{\pi \text{ rad}}{180°}\right) = \frac{\pi}{20}\text{ rad}$

Clave E

Nivel 2 (página 13) Unidad 1

Comunicación matemática

17. Por teoría : I. a
II. c
III. b

Clave D

18. Por teoría : I. (V)
II. (F)
III. (V)

Clave E

Razonamiento y demostración

19. $K = \frac{1°2'}{2'} + \frac{1°3'}{3'} + \frac{1°4'}{4'}$; como $1° = 60'$

$K = \frac{60'+2'}{2'} + \frac{60'+3'}{3'} + \frac{60'+4'}{4'}$

$K = \frac{62'}{2'} + \frac{63'}{3'} + \frac{64'}{4'}$

$K = 31 + 21 + 16$

$\therefore\ K = 68$

Clave E

20. $67°\ 30' = 67° + \frac{1°}{2} = \frac{135°}{2}\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{3\pi}{8}\text{ rad}$

Clave B

21. $P = 40^g + \frac{3\pi}{4}\text{ rad}$

$P = 40^g\left(\frac{9°}{10^g}\right) + \frac{3\pi}{4}\text{rad}\left(\frac{180°}{\pi \text{ rad}}\right)$

$\therefore\ P = 36° + 135° = 171°$

Clave A

22. $J = \sqrt{\frac{C^2+S^2}{C^2-S^2} - \frac{10}{19}} = \sqrt{\frac{(10k)^2+(9k)^2}{(10k)^2-(9k)^2} - \frac{10}{19}}$

$J = \sqrt{\frac{100k^2+81k^2}{100k^2-81k^2} - \frac{10}{19}} = \sqrt{\frac{181k^2}{19k^2} - \frac{10}{19}}$

$J = \sqrt{\frac{171}{19}}$

$\therefore\ J = \sqrt{9} = 3$

Clave C

23. $J = \frac{\pi C - 60R}{\pi S - 40R}$

Usamos:

$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi} = k$

$J = \frac{\pi(200k) - 60(\pi k)}{\pi(180k) - 40(\pi k)} = \frac{200\pi k - 60\pi k}{180\pi k - 40\pi k}$

$\therefore\ J = \frac{140\pi k}{140\pi k} = 1$

Clave A

24. $F = \frac{405 \cdot (C-S)^3}{S^2 \cdot C}$

Usamos la relación entre S y C:

$\dfrac{S}{9} = \dfrac{C}{10} = k \Rightarrow S = 9k \ \wedge \ C = 10k$

$F = \dfrac{405(10k - 9k)^3}{(9k)^2(10k)} = \dfrac{405 \,.\, k^3}{81k^2 \,.\, 10k}$

$F = \dfrac{405k^3}{810k^3}$

$\therefore F = \dfrac{405}{810} = \dfrac{1}{2}$

Clave C

Resolución de problemas

25. Un ángulo mide $30^g \left(\dfrac{9°}{10^g}\right) = \dfrac{30\,.\,9°}{10} = 27°$

Su complemento será: $90° - 27° = 63°$

Por dato: $(8x - 1)° = 63° \Rightarrow 8x = 64$

$\therefore x = 8$

Clave D

26. $3C - 2S = 36$; usando la relación entre S y C.

$3\left(\dfrac{10S}{9}\right) - 2S = 36 \Rightarrow \dfrac{30}{9}S - 2S = 36$

$\Rightarrow S = 27°$

La medida circular será:

$27°\left(\dfrac{\pi rad}{180°}\right) = \dfrac{3\pi}{20}$ rad

Clave C

27. Del enunciado: $C - \dfrac{S}{3} = 28$

Usando la relación entre S y C:

$\left(\dfrac{10}{9}S\right) - \dfrac{S}{3} = 28$

$\dfrac{7S}{9} = 28 \Rightarrow S = 36$

$\Rightarrow$ El ángulo mide 36°.

La medida circular será:

$36°\left(\dfrac{\pi rad}{180°}\right) = \dfrac{\pi}{5}$ rad

Clave B

28. $S = 2n + 1 \ \wedge \ C = 3n - 2$

$\left(\dfrac{S}{C}\right) = \dfrac{2n+1}{3n-2} \Rightarrow \left(\dfrac{9}{10}\right) = \dfrac{2n+1}{3n-2}$

$\Rightarrow 27n - 18 = 20n + 10$

$7n = 28 \Rightarrow n = 4$

Reemplazando el valor de n en S:

$S = 2(4) + 1 = 9$

Entonces el ángulo mide 9°, en la medida circular será:

$9°\,.\left(\dfrac{\pi\ rad}{180°}\right) = \dfrac{\pi}{20}$ rad

Clave C

29. Sean los ángulos α y β.

Del enunciado $\begin{cases} \alpha + \beta = 90° \\ \alpha - \beta = \dfrac{\pi}{10}\text{ rad} = 18° \end{cases}$

Resolviendo: $\alpha = 54° \ \wedge \ \beta = 36°$

$\therefore$ El mayor mide 54°.

Clave B

30. $SC = 810$; usando la relación entre S y C:

$S\left(\dfrac{10}{9}S\right) = 810$

$\dfrac{10}{9}S^2 = 810 \Rightarrow S^2 = 729$

$S = 27$

$\Rightarrow$ El ángulo mide 27°.

La medida circular será:

$27°\left(\dfrac{\pi\ rad}{180°}\right) = \dfrac{3\pi}{20}$ rad

Clave B

31. Sean los ángulos α y θ.

Del enunciado $\begin{cases} \alpha + \theta = 180° \\ \alpha - \theta = \dfrac{\pi}{3}\text{ rad} = 60° \end{cases}$

Resolviendo: $\alpha = 120° \ \wedge \ \theta = 60°$

$\therefore$ El menor ángulo mide 60°.

Clave D

Nivel 3 (página 14) Unidad 1

Comunicación matemática

32.

I. Del gráfico se observa que el ángulo divide a una vuelta en tres partes, entonces:

Sea θ el ángulo; $\theta = \dfrac{m\angle 1\text{ vuelta}}{3}$

$\therefore$ I (V)

II. Ya que el ángulo es la tercera parte de una vuelta:

$\theta = \dfrac{m\angle 1\text{ vuelta}}{3} = \dfrac{2\pi\text{ rad}}{3}$

Luego, θ es igual a $\dfrac{2\pi}{3}$ rad en el sistema internacional.

$\therefore$ II (F)

III. Análogamente:

$\theta = \dfrac{m\angle 1\text{ vuelta}}{3} = \dfrac{400^g}{3}$

Luego, θ es igual a $\dfrac{400^g}{3}$ en el sistema centesimal. $\therefore$ III (V)

Clave A

33. Los sistemas centesimal y sexagesimal hacen uso de subunidades las cuales están definidas:

$1° = 60' \qquad \wedge \qquad 1^g = 100^m$

$1' = 60'' \qquad\qquad\quad 1^m = 100^s$

Comparando con las expresiones anteriores:

$1° = 60'$, entonces: $1' = \dfrac{1°}{60}$

Clave D

Razonamiento y demostración

34. $3'\,7'' = 3'\,.\left(\dfrac{60''}{1'}\right) + 7'' = 180'' + 7'' = 187''$

Clave A

35. $22°\,30' = 22° + 30'\,.\left(\dfrac{1°}{60'}\right)$

$22° + \dfrac{1°}{2} = \dfrac{45°}{2}$

$\dfrac{45°}{2}\left(\dfrac{\pi rad}{180°}\right) = \dfrac{\pi}{8}$ rad

Clave D

36. En el mismo sentido antihorario:

$x° + (-y^g) = 45°$

$x° - y^g\,.\left(\dfrac{9°}{10^g}\right) = 45°$

$x - \dfrac{9y}{10} = 45$

$10x - 9y = 450$

Clave B

37. Usando la relación entre S y C: $\dfrac{S}{9} = \dfrac{C}{10} = k$, tenemos que la expresión es:

$J = \sqrt{\dfrac{2(10k) - (9k)}{(10k) - (9k)} + \sqrt{\dfrac{5(9k) - 2(10k)}{(10k) - (9k)}}}$

Eliminando la constante k y reduciendo tenemos:

$J = \sqrt{11 + \sqrt{25}} = \sqrt{11 + 5} = \sqrt{16} = 4$

Clave B

Resolución de problemas

38. Del enunciado:

$\sqrt{\left(\dfrac{C}{2}\right)(3S)} = 6\sqrt{15} \quad \Rightarrow \dfrac{3S\,.\,C}{2} = 540$

Usando la relación entre S y C:

$\dfrac{3}{2}\,.\,S\,.\left(\dfrac{10}{9}S\right) = 540 \Rightarrow S^2 = 324$

$S = 18$

$\Rightarrow$ El ángulo mide 18°.

La medida circular será:

$18°\left(\dfrac{\pi rad}{180°}\right) = \dfrac{\pi}{10}$ rad

Clave A

39. Del enunciado: $C - S = 3$

Usando la relación entre S y C:

$\left(\dfrac{10}{9}S\right) - S = 3$

$\dfrac{10S - 9S}{9} = 3 \Rightarrow \dfrac{S}{9} = 3 \Rightarrow S = 27$

$\Rightarrow$ El ángulo mide 27°.

La medida circular será:

$27°\,.\left(\dfrac{\pi rad}{180°}\right) = \dfrac{3\pi}{20}$ rad

Clave D

40. Usamos la relación entre S, C y R:

$\dfrac{S}{180} = \dfrac{C}{200} = \dfrac{R}{\pi} = k$

Reemplazamos en la expresión:

$2\,.\sqrt{\dfrac{\pi}{\pi k}} - 3\sqrt{\dfrac{\pi k}{\pi}} = 2\sqrt{2}$

$2\,.\sqrt{\dfrac{1}{k}} - 3\sqrt{k} = 2\sqrt{2}$

Si $\sqrt{k} = a$, entonces: $(a > 0)$

$\dfrac{2}{a} - 3a = 2\sqrt{2}$

$3a^2 + 2\sqrt{2}\,a - 2 = 0$

$3a \qquad -\sqrt{2}$

$a \qquad +\sqrt{2} \Rightarrow a = -\sqrt{2}$ (no cumple)

$\sqrt{k} = \dfrac{\sqrt{2}}{3} \Rightarrow k = \dfrac{2}{9}$

$\Rightarrow S = 180k = 180\left(\dfrac{2}{9}\right) = 40$

$\therefore$ La medida del ángulo es 40°.

Clave C

APLICAMOS LO APRENDIDO (página 15) Unidad 1

1. $\theta = 120° . \left(\frac{\pi \text{ rad}}{180°}\right) = \frac{2\pi}{3}$ rad

$R = 12$ cm

$L = \theta . R$

$L = \frac{2\pi}{3} . 12$

$L = 8\pi$ cm

Clave A

2. $\theta = 62^g . \left(\frac{\pi \text{ rad}}{200^g}\right) = \frac{31\pi}{100}$ rad

$R = 1$ m $= 100$ cm

$L = \theta . R$

$L = \frac{31\pi}{100} . 100$

$L = 31\pi$ cm

Clave D

3. $\theta = \frac{\pi}{5}$ rad

$R = 5$ m

$L = \theta . R$

$L = \frac{\pi}{5} . 5$

$L = \pi$ m

Clave A

4. $\theta = x$ rad

$R = 5$

$L = 3x + 4$

$L = \theta . R$

$3x + 4 = (x) . (5)$

$3x + 4 = 5x$

$x = 2$

Clave B

5. $\theta = 28°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{7\pi}{45}$ rad

$R = 15$

$L = \theta . R$

$L = \frac{7\pi}{45} . 15$

$L = \frac{7\pi}{3}$

Clave C

6. $\theta = 40^g . \left(\frac{\pi \text{ rad}}{200^g}\right) = \frac{\pi}{5}$ rad

$R = 15$

$L = \theta . R$

$L = \frac{\pi}{5} . 15$

$L = 3\pi$

Clave E

7. $\theta = \frac{\pi}{7}$ rad

$R = 35$

$L = \theta . R$

$L = \frac{\pi}{7} . 35$

$L = 5\pi$

Clave A

8. Del enunciado:

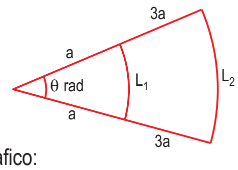
$L = 3R$

$\theta . R = 3R$

$\theta = 3$ rad

Clave C

9.



Del gráfico:

$L_1 = \theta . a$ ...(I)

$L_2 = \theta . (4a)$ ...(II)

Dividiendo (II) entre (I):

$\frac{L_2}{L_1} = \frac{4\theta a}{\theta a} = 4$

Clave A

10. $\theta = \frac{\pi}{5}$ rad

$L = 3\pi$ m

$R = ?$

$L = \theta . R$

$3\pi = \frac{\pi}{5} . R \Rightarrow R = \frac{15\pi}{\pi} = 15$

$\therefore$ El radio mide 15 m.

Clave D

11. Sabemos:

$L = \theta . R$

$6 = \alpha . 30$

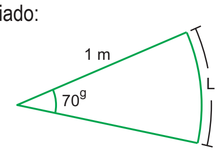
$1 = \alpha . 5$

$\alpha = \frac{1}{5}$

$\therefore \ \alpha = 0{,}2$ rad

Clave C

12. Del enunciado:



Expresamos el ángulo en radianes:

$70^g = 70^g . \frac{\pi \text{ rad}}{200^g} = \frac{7\pi}{20}$ rad

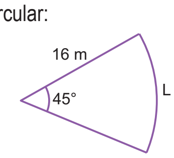
Sabemos:

$L = \theta . R = \frac{7\pi}{20}$. (1)

$L = \frac{7\pi}{20}$ $\quad \therefore L = \frac{7\pi}{20}$ m

Clave A

13. Del sector circular:



Transformamos el ángulo a radianes:

$45° = 45° . \frac{\pi \text{ rad}}{180°} = \frac{\pi}{4}$ rad

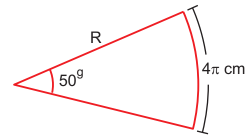
$45° = \frac{\pi}{4}$ rad

Sabemos:

$L = \theta . R = \frac{\pi}{4} 16 = 4\pi$ $\quad \therefore$ $L = 4\pi$ m

Clave D

14. Del enunciado:



Transformamos el ángulo a radianes:

$50^g = 50^g . \frac{\pi \text{ rad}}{200^g} = \frac{\pi}{4}$ rad

Sabemos:

$L = \theta . R$

$4\pi = \frac{\pi}{4} . R$ $\quad \therefore R = 16$ cm

Clave C

PRACTIQUEMOS

Nivel 1 (página 17) Unidad 1

Comunicación matemática

1. Por la definición del cálculo de longitud de arco:

$\alpha R = L$

I. α representa el número de radianes del ángulo central.
$\therefore$ I es falsa.

II. Para el cálculo de longitud de arco, las unidades se determinan con las unidades del radio, α solo es un número (número de radianes del ángulo).
$\therefore$ II es falsa.

III. De la expresión $\alpha R = L$, si R es igual a L entonces:
$\alpha R = R$
$\alpha = 1$
Por lo tanto, el ángulo central es igual a 1 rad. (α indica el número de radianes del ángulo).
$\therefore$ III es falsa.

Clave B

2. De la expresión para el cálculo de una longitud de área, en el gráfico:

$\theta R_1 = a, \theta R_2 = b$ ... (1):

Además, por propiedad del trapecio circular:

$\theta = \frac{b - a}{R_2 - R_1}$ ... (2)

Finalmente, de (1):

$\theta = \frac{a}{R_1} = \frac{b}{R_2}$; $R_2 = \frac{b}{\theta}$

De 2:

$b - a = \theta(R_2 - R_1)$

Clave A

Razonamiento y demostración

3. $\theta = 2$ rad

$R = 3$ m

$L = \theta . R$

$L = 2 . 3 \Rightarrow L = 6$ m

Clave D

4. $\theta = ?$

$R = 6$ m

$L = \frac{3\pi}{4}$ m

$L = \theta . R$

$\frac{3\pi}{4} = \theta . 6$

$\theta = \frac{\pi}{8}$ rad

Clave C

5. Del gráfico:

$L = 6\pi$ m

$\theta = \frac{\pi}{3}$ rad

Entonces:

$L = \theta \times R$

$6\pi = \frac{\pi}{3} \times R$

$R = 18$ m

Clave D

6. Del gráfico:

$L = 24$ m

$R = 8$ m

Entonces:

$L = \theta \times R$

$24 \text{ m} = \theta \times 8 \text{ m}$

$\theta = 3$ rad

Clave C

7. Del gráfico:

$R = 30$ m

$L = 6$ m

Entonces:

$L = \theta \times R$

$6 \text{ m} = \theta \times 30 \text{ m}$

$\theta = \frac{1}{5} \text{ rad} = 0{,}2 \text{ rad}$

Clave D

8. $\theta = 60° = \frac{\pi}{3}$ rad

$L = 4$ m

$L = \theta \, . \, r$

$4 = \frac{\pi}{3} \, . \, r \Rightarrow r = \frac{12}{\pi}$ m

Clave E

Resolución de problemas

9. $\theta = 0{,}5$ rad

$R = 4$ m

$L = \theta \, . \, R$

$L = (0{,}5)(4) = 2$ m

Nos piden el perímetro del sector circular:

Perímetro $= 2R + L = 2(4) + (2) = 10$ m

Clave D

10. Del problema:

$R = 24$ m

$\theta = \frac{2}{3}$ rad

Piden L:

$L = (24 \text{ m})\left(\frac{2}{3}\right)$

$L = 16$ m

Clave D

Nivel 2 (página 18) Unidad 1

Comunicación matemática

11.

I. La longitud de arco de un ángulo central está definido como el producto del radio y el número de radianes del ángulo central.

Para el gráfico

Ángulo central: $\theta°$

Luego:

$\theta° = \theta° \, . \, \frac{\pi}{180°}$ rad

$\theta° = \frac{\theta\pi}{180}$ rad

Por lo tanto:

$L = \left(\frac{\theta\pi}{180}\right) . R$ I es falso

II. $\theta°$ es la medida del ángulo central en el sistema sexagesimal entonces θ indica el número de grados sexagesimales.

II es verdadera

III. De la relación:

$\frac{\pi\theta}{180} \, . \, R = L$

Por dato, R se encuentra en metros. Por lo tanto, L también será calculada en metros, además:

$\frac{L}{R} = \underbrace{\frac{\theta\pi}{180}}_{\text{n.° de radianes del ángulo central}}$ III es verdadera

Clave A

12.

I. $L = \frac{\pi}{2} \, . \, 2$; radio en metros

$L = \pi$ m

II. $L = \pi \, . \, 1$; radio en centímetros

$L = \pi$ cm

III. $L = \frac{\pi}{4} \, . \, 1$; radio en metros

$L = \frac{\pi}{4}$ m

Clave D

Razonamiento y demostración

13. $\theta = 60° = \frac{\pi}{3}$ rad

$R = 6$ m

$L = x$

$L = \theta \, . \, R$

$x = \frac{\pi}{3} \, . \, 6$

$x = 2\pi$ m

Clave D

14. $\theta = 135° = \frac{3\pi}{4}$ rad

$R = 8$ m

$L = x$

$L = \theta \, . \, R$

$x = \frac{3\pi}{4} \, . \, 8$

$x = 6\pi$ m

Clave C

15. $\theta = 50^g \, . \left(\frac{\pi \text{ rad}}{200^g}\right) = \frac{\pi}{4}$ rad

$R = 2$ m

$L = \theta \, . \, R$

$L = \frac{\pi}{4} \, . \, 2$

$L = \frac{\pi}{2}$ m

Clave B

16. $\theta = 108° = \frac{3\pi}{5}$ rad

$L = 2\pi$ m

$L = \theta \, . \, R$

$2\pi = \frac{3\pi}{5} \, . \, R$

$R = \frac{10\pi}{3\pi} \Rightarrow R = \frac{10}{3}$ m

Clave E

17. $\theta = 30° = \frac{\pi}{6}$ rad

$R = 3$ m

$L = \theta \, . \, R$

$L = \frac{\pi}{6} \, . \, 3$

$L = \frac{\pi}{2}$ m

Clave E

18. $\theta = 120° \, . \left(\frac{\pi \text{ rad}}{180°}\right) = \frac{2\pi}{3}$ rad

$R = 3$ m

$L = x$

$L = \theta \, . \, R$

$x = \frac{2\pi}{3} \, . \, 3$

$x = 2\pi$ m

Clave D

Resolución de problemas

19. Del problema:

$R = 30$ m

$L = 20$ m

Pide θ:

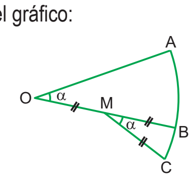
$L = \theta \times R$

$20 \text{ m} = \theta \times 30 \text{ m}$

$\theta = \frac{2}{3}$ rad

Clave B

20. Del gráfico:



$\alpha = \frac{\pi}{6}$ rad

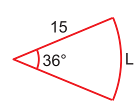
$\left.\begin{aligned} L_{\widehat{AB}} &= \frac{\pi}{6}(4R) \\ L_{\widehat{CB}} &= \frac{\pi}{6}(2R) \end{aligned}\right\} (+)$

$L_{\widehat{AB}} + L_{\widehat{CB}} = \frac{2\pi}{3}R + \frac{\pi}{3}R$

$L_{\widehat{AB}} + L_{\widehat{CB}} = \pi R$

Clave A

21.



36° a radianes:

$\frac{S}{9} = \frac{20R}{\pi}$

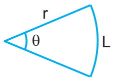
$\frac{36}{9} = \frac{20R}{\pi}$

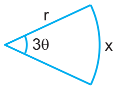
$R = \frac{\pi}{5} \Rightarrow \theta = \frac{\pi}{5}$ rad

$L = \frac{\pi}{5} \, . \, 15$

$\therefore L = 3\pi$ cm

Clave C

22. 

$\Rightarrow$ 

$\theta \,.\, r = L$ $\qquad x = 3\theta \,.\, r$

$\therefore\ x = 3L$

Clave B

Nivel 3 (página 19) Unidad 1

Comunicación matemática

23. De la relación: $\theta R = L$

I. De la figura

$\theta(20) = 5\pi$

$\theta = \dfrac{\pi}{4}$

$\angle AOB$ es agudo.

II. Análogamente en la figura:

$\theta(18) = 15\pi$

$\theta = \dfrac{5}{6}\pi$

$\therefore\ \angle AOB$ es $\dfrac{5}{6}\pi$ rad.

III. De la figura, para calcular, R y L deben estar en el mismo sistema de unidades (cm).

$\theta(180) = 90\pi$

$\theta = \dfrac{\pi}{2}$

$\therefore\ \angle AOB$ es recto.

Clave E

24.

I. En la figura:

$\theta R_1 = L_1 \quad \wedge \quad \theta R_2 = L_2$

$\theta = \dfrac{L_1}{R_1}$ ... (1); $\quad \theta = \dfrac{L_2}{R_2}$ ... (2)

De (1) y (2)

$\dfrac{L_1}{R_1} = \dfrac{L_2}{R_2}$

$L_1R_2 = L_2R_1$

$\therefore$ I es verdadera

II. Por propiedad:

$\theta = \dfrac{L_2 - L_1}{h}$

$\therefore$ II es falsa

III. En la figura:

$\theta R_2 = L_2$ ... (1)

Además:

$R_2 = R_1 + h$ ... (2)

(2) en (1):

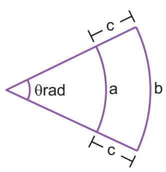
$L_2 = \theta(R_1 + h)$

$\therefore$ III es verdadera

Clave: A

Razonamiento y demostración

25. Sabemos:



$\theta = \dfrac{b - a}{c}$

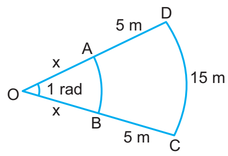
En el gráfico del ejercicio:

$b = 4,\ a = 3,\ c = 2$

$\theta = \dfrac{4 - 3}{2} = \dfrac{1}{2}$

Clave E

26. Del gráfico:



$L_{\widehat{AB}} = 1(x) = x$

$L_{\widehat{DC}} = 15 = 1(5 + x) \Rightarrow x = 10$ m

Clave C

Resolución de problemas

27. $\theta = a$ rad

$R = a + 1$

$L = a + 4$

$L = \theta \,.\, R$

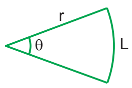
$(a + 4) = a(a + 1)$

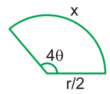
$0 = a^2 - 4$

$0 = (a + 2)(a - 2)\ \begin{cases} a = 2 \\ a = -2 \end{cases}$

Como a es un ángulo y (a + 1) es el radio $\Rightarrow$ a = 2

Clave A

28. 

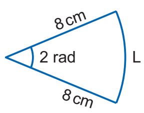
$\Rightarrow$ 

$\theta \,.\, r = L$ $\qquad x = 4\theta \,.\, \dfrac{r}{2}$

$x = 2\theta r = 2(L)$

$x = 2L$

Clave E

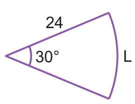
29. 

$L = 2 \,.\, 8 = 16$

Por lo tanto:

perímetro: 16 + 8 + 8 = 32 cm

Clave B

30. 

30° a radianes:

$\dfrac{S}{9} = \dfrac{20R}{\pi}$

$\dfrac{30}{9} = \dfrac{20R}{\pi}$

$R = \pi/6 \Rightarrow \theta = \dfrac{\pi}{6}$ rad

$L = \dfrac{\pi}{6} \,.\, 24 = 4\pi$ cm

Clave D

MARATÓN MATEMÁTICA (página 20)

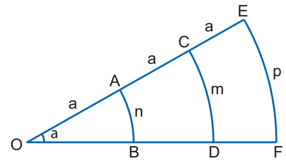
1. $H = \dfrac{(S + R)^2 - (S - R)^2}{(C + R)^2 - (C - R)^2}$

$H = \dfrac{S^2 + 2RS + R^2 - (S^2 - 2SR + R^2)}{C^2 + 2CR + R^2 - (C^2 - 2CR + R^2)}$

$H = \dfrac{S^2 + 2RS + R^2 - S^2 + 2SR - R^2}{C^2 + 2CR + R^2 - C^2 + 2CR - R^2}$

$H = \dfrac{4RS}{4CR} = \dfrac{S}{C} = \dfrac{9}{10} = 0{,}9$

Clave B

2. 

$\lhd AOB: = n = \alpha(a)$ ...(1)

$\lhd COD: = m = \alpha(2a)$ ...(2)

$\lhd EOF: = p = \alpha(3a)$ ...(3)

$(1) + (2) \Rightarrow m + n = \underbrace{3a\alpha}_{P}$

$\therefore P = m + n$

Clave C

3. Dado el ángulo: $(4a + 11)°$ y $(12a - 18)^g$

Por ser ángulos equivalentes se cumple:

$\dfrac{S}{9} = \dfrac{C}{10} \Rightarrow \dfrac{4a + 11}{9} = \dfrac{12a - 18}{10}$

$10(4a + 11) = 9(12a - 18)$

$40a + 110 = 108a - 162$

$68a = 272$

$a = 4$

Luego el ángulo es:

$(4a + 11)° = (4(4) + 11)° = 27°$

$(12a - 18)^g = (12(4) - 18)^g = 30^g$

El ángulo representado en radianes será:

$\dfrac{30}{200} = \dfrac{R}{\pi} \Rightarrow R = \dfrac{3\pi}{20}$

Clave D

4. Sea el ángulo x:

$S(C(S(C(x)))) = 190°$

$S(C(S(90° - x))) = 190°$

$S(C(180° - 90° + x)) = 190°$

$S(C(90° + x)) = 190°$

$S(90 - 90 - x) = 190°$

$S(-x) = 190°$

$180 - (-x) = 190°$

$180 + x = 190°$

$x = 10°$

Nos piden calcular el suplemento del ángulo aumentado en 20^g.

$\dfrac{S}{9} = \dfrac{20}{10} \Rightarrow S = \dfrac{20 \,.\, 9}{10} = 18°$

Luego:

$S(10° + 18°) = S(28°) = 180° - 28° = 152°$

Clave E

5. Sabemos: $\dfrac{S}{9} = \dfrac{C}{10} = k \Rightarrow S = 9k$

$C = 10k$

Luego:

$3S - 2C = 84$

$3(9k) - 2(10k) = 84$

$27k - 20k = 84$

$7k = 84$

$k = 12$

El ángulo en grados sexagesimales es:
$S = 9k = 9(12) = 108°$

El suplemento es: $180° - 108° = 72°$

Lo convertimos en grados centesimales:

$$\frac{72}{9} = \frac{C}{10} \Rightarrow C = \frac{72 \, . \, 10}{9} = 80^g$$

Clave C

6. Sean los ángulos:
$\alpha \wedge \beta; \alpha > \beta$
Del enunciado tenemos:

$$\left.\begin{aligned} \alpha + \beta &= \frac{3}{5}\pi \\ \alpha - \beta &= 20^g \end{aligned}\right\} (+)$$

$$\Rightarrow 2\alpha = \underbrace{\frac{3}{5}\pi}_{S_1} + \underbrace{20^g}_{S_2}$$

Convertimos cada una de las medidas al sistema sexagesimal.

$$\frac{S_1}{180} = \frac{3\pi}{5\pi} \Rightarrow S_1 = 108°$$

$$\frac{S_2}{9} = \frac{20}{10} \Rightarrow S_2 = 18°$$

Reemplazamos:

$$2\alpha = 108° + 18°$$
$$2\alpha = 126° \Rightarrow \alpha = 63°$$

Clave E

7. Del enunciado:

$$90° - 30^g = \frac{b\pi}{20}$$

Realizamos las conversiones:

$$30^g = 27° \;\wedge\; \frac{b\pi}{20} = (9b)°$$

Luego:

$$90° - 27° = (9b)°$$
$$63 = 9b$$
$$7 = b$$

Los ángulos son:
27° y 63°
$\Rightarrow 63° - 27° = 36°$

Clave C

8. Del gráfico:

$$\frac{\pi}{6} - 2x - (20^g - 8x) = 90°$$

Realizamos las conversiones:

$$\frac{S}{180} = \frac{\frac{\pi}{6}}{\pi} \Rightarrow S = \frac{180}{6} = 30°$$

$$\frac{S}{9} = \frac{20}{10} \Rightarrow S = 18°$$

Luego:

$$30° - 2x - (18° - 8x) = 90°$$
$$30° - 2x - 18° + 8x = 90°$$
$$6x = 78°$$
$$x = 13°$$

Por último:
$C(13°) = 90° - 13° = 77°$

Clave B

9. Convertimos el ángulo 40^g a radianes:

$$\frac{R}{\pi} = \frac{40}{200} \Rightarrow R = \frac{\pi}{5}$$

Sabemos: $L = \theta \, . \, R$

$$\left(\frac{x}{3} - \frac{4}{5}\right)\pi = \frac{\pi}{5} \, . \, x$$

$$\frac{5x - 12}{15} = \frac{x}{5}$$

$$5x - 12 = 3x$$
$$2x = 12$$
$$x = 6$$

Clave C

ÁREA DEL SECTOR CIRCULAR

APLICAMOS LO APRENDIDO (página 23) Unidad 2

1. $\left\{\begin{array}{l}\theta = 40^g\left(\frac{\pi \text{ rad}}{200^g}\right) = \frac{\pi}{5} \text{ rad} \\ R = 10\text{m}\end{array}\right\}$ Área del sector circular $S = \frac{\theta . R^2}{2}$

$S = \dfrac{\left(\frac{\pi}{5}\right)(10)^2}{2} = 10\pi \text{ m}^2$

Clave C

2. $\left.\begin{array}{l}L = 3 \text{ m} \\ R = 2 \text{ m}\end{array}\right\}$ Área del sector circular: $S = \frac{L.R}{2}$

$S = \dfrac{(3)(2)}{2} = 3 \text{ m}^2$

Clave E

3. $\left.\begin{array}{l}\theta = 1 \text{ rad} \\ R = 4 \text{ m}\end{array}\right\}$ Área del sector circular: $S = \frac{\theta . R^2}{2}$

$S = \dfrac{(1)(4)^2}{2} = 8 \text{ m}^2$

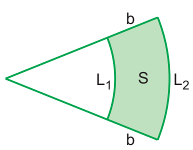
Clave C

4. $\left.\begin{array}{l}\theta = 0{,}5 \text{ rad} \\ L = 2 \text{ m}\end{array}\right\}$ Área del sector circular: $S = \frac{L^2}{2\theta}$

$S = \dfrac{(2)^2}{2(0{,}5)} = 4 \text{ m}^2$

Clave B

5.



El área de un trapecio circular se puede calcular como:

$S = \left(\dfrac{L_1 + L_2}{2}\right)b$

$\Rightarrow$ Para: $L_1 = 6$ m

$L_2 = 8$ m

$b = 4$ m

$S = \dfrac{(6+8)}{2}.4 = 28 \text{ m}^2$

Clave B

6. $\left.\begin{array}{l}\theta = 120° = \frac{2\pi}{3} \text{ rad} \\ R = 6 \text{ m}\end{array}\right\}$ Área del sector circular: $S = \frac{\theta . R^2}{2}$

$S = \dfrac{\left(\frac{2\pi}{3}\right)(6)^2}{2} = 12\pi \text{ m}^2$

Clave B

7. $\left.\begin{array}{l}\theta = \frac{\pi}{8} \text{ rad} \\ R = 2 \text{ m}\end{array}\right\}$ Área del sector circular: $S = \frac{\theta . R^2}{2}$

$S = \dfrac{\left(\frac{\pi}{8}\right)(2)^2}{2} = \dfrac{\pi}{4} \text{ m}^2$

Clave E

8. $\left.\begin{array}{l}\theta = 50^g\left(\frac{\pi \text{ rad}}{200^g}\right) = \frac{\pi}{4} \text{ rad} \\ L = \sqrt{2\pi} \text{ m}\end{array}\right\}$ Área del sector circular: $S = \frac{L^2}{2\theta}$

$S = \dfrac{(\sqrt{2\pi})^2}{2\left(\frac{\pi}{4}\right)} = 4 \text{ m}^2$

Clave A

9. Del enunciado:

Área del sector circular: $S = 4 \text{ m}^2$

El perímetro del sector circular: $P = 8$ m

$P = 8$

$2R + L = 8$ ...(I)

$S = 4$

$\dfrac{L.R}{2} = 4 \Rightarrow L.R = 8$ ...(II)

Reemplazando (I) en (II):

$(8 - 2R)(R) = 8$

$8R - 2R^2 = 8 \Rightarrow \underbrace{R^2 - 4R + 4}_{(R-2)^2} = 0$

$(R - 2)^2 = 0$

$R = 2$ m

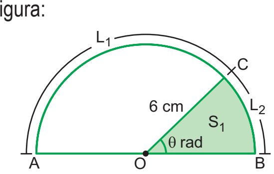
Clave B

10. $\left.\begin{array}{l}\theta = 40°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{2\pi}{9} \text{ rad} \\ L = 4 \text{ m}\end{array}\right\}$ Área del sector circular: $S = \frac{L^2}{2\theta}$

$S = \dfrac{(4)^2}{2\left(\frac{2\pi}{9}\right)} = \dfrac{16}{\frac{4\pi}{9}} = \dfrac{16.9}{4\pi} = \dfrac{36}{\pi} \text{ m}^2$

Clave B

11. De la figura:



$L_1 = (\pi - \theta).6 = 6(\pi - \theta)$

$L_2 = \theta . 6 = 6\theta$

Por dato:

$L_1 = 8L_2$

$6(\pi - \theta) = 8(6\theta)$

$\pi - \theta = 8\theta$

$9\theta = \pi$

$\theta = \pi/9$

$S_1 = \dfrac{1}{2}\theta(6)^2$

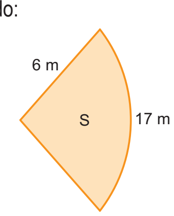
$S_1 = \dfrac{1}{2}.\dfrac{\pi}{9}(6)^2 = \dfrac{\pi}{\cancel{18}}.\cancel{36} = 2\pi$

$S_1 = 2\pi$

$\therefore S_1 = 2\pi \text{ cm}^2$

Clave B

12. Del enunciado:



Sabemos:

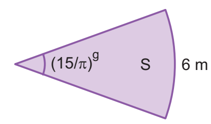
$S = \dfrac{L R}{2}$

$S = \dfrac{17.\cancel{6}}{\cancel{2}} = 51$

$\therefore S = 51 \text{ m}^2$

Clave D

13. Del enunciado:



Transformamos el ángulo a radianes:

$\dfrac{15^g}{\pi} = \dfrac{15^g}{\pi}.\dfrac{\pi \text{ rad}}{200^g} = \dfrac{3}{40} \text{ rad}$

Sabemos:

$S = \dfrac{L^2}{2\theta}$

$S = \dfrac{(6)^2}{2\left(\frac{3}{40}\right)} = \dfrac{36.40}{6} = 240$

$S = 240$

$\therefore S = 240 \text{ m}^2$

Clave A

14. Por propiedad de trapecios circulares:

$S = \dfrac{(a+b)}{2}$ ... (1)

Por dato:

$a = 7$ m; $b = 19$ m; $S = 39 \text{ m}^2$; $c = x$

Reemplazando en (1)

$39 = \dfrac{(7+19)}{2}x$

$39 = \dfrac{26}{2}x$

$13x = 39$

$x = 3$

$\therefore x = 3$ m

Clave B

PRACTIQUEMOS
Nivel 1 (página 25) Unidad 2

Comunicación matemática

1.

I. La definición pertenece a la circunferencia. I es falsa.

II. Para un círculo, su ángulo central es igual a 2π; reemplazando en la expresión:

$S = \dfrac{1}{2}\theta R^2 = \dfrac{1}{2}(2\pi)R^2 = \pi R^2$

$S = \pi R^2$

Donde S: área del círculo.

II es falsa.

III. De la expresión:

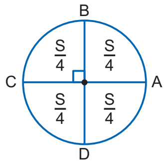
$S = \dfrac{1}{2}\theta R^2$

Las unidades de S están determinadas por las unidades del radio al cuadrado. Por lo tanto, si R está en metros (m), S tendrá como unidad al metro cuadrado (m^2).

III es verdadera.

Clave C

2. De la circunferencia C_1:



I. De la figura, se observa que en I está sombreada la cuarta parte del círculo; luego:
1 círculo → S
1/4 círculo → x
x = 1/4 . S
x = S/4

II. La figura indica la mitad del círculo sombreada, entonces:
1 círculo → S
1/2 círculo → y
y = 1/2 . S
y = S/2

III. La figura muestra los 3/4 del círculo sombreado; luego:
1 círculo → S
3/4 círculo → z
z = 3/4 . S
$z = \frac{3S}{4}$

Clave B

Razonamiento y demostración

3. Área del sector circular: $S = \frac{\theta . R^2}{2}$

$$\left.\begin{array}{l}\theta = \frac{\pi}{5}\ \text{rad} \\ R = 20\ \text{cm}\end{array}\right\} \Rightarrow S = \frac{\left(\frac{\pi}{5}\right)(20)^2}{2} = 40\pi$$

Clave E

4. Área del sector circular: $S = \frac{\theta . R^2}{2}$

$$\left.\begin{array}{l}\theta = \frac{\pi}{7}\ \text{rad} \\ R = 2\sqrt{7}\ \text{m}\end{array}\right\} \Rightarrow S = \frac{\left(\frac{\pi}{7}\right)(2\sqrt{7})^2}{2} = 2\pi$$

Clave D

5. Área del sector circular: $S = \frac{\theta . R^2}{2}$

$$\left.\begin{array}{l}\theta = \frac{\pi}{9}\ \text{rad} \\ R = 27\ \text{m}\end{array}\right\} \Rightarrow S = \frac{\left(\frac{\pi}{9}\right)(27)^2}{2} = \frac{81}{2}\pi$$

Clave D

6. Área del sector circular: $S = \frac{L . R}{2}$

$$\left.\begin{array}{l}L = 8\ \text{m} \\ R = 4\ \text{m}\end{array}\right\} \Rightarrow S = \frac{(8)(4)}{2} = 16\ \text{m}^2$$

Clave C

7. Área del sector circular: $S = \frac{L . R}{2}$

$$\left.\begin{array}{l}L = 6b \\ R = 2a\end{array}\right\} \Rightarrow S = \frac{(6b)(2a)}{2} = 6ab$$

Clave E

8. $\left.\begin{array}{l}\theta = 0{,}5\ \text{rad} \\ L = \sqrt{13}\ \text{m}\end{array}\right\}$ Área del sector circular:

$$S = \frac{L^2}{2 . \theta} = \frac{(\sqrt{13})^2}{2(0{,}5)} = 13\ \text{m}^2$$

Clave D

9. Área del sector circular:

$S = \frac{L . R}{2}$

$$\left.\begin{array}{l}R = 18\ \text{m} \\ L = 15\ \text{m}\end{array}\right\} \Rightarrow S = \frac{(15)(18)}{2} = 135\ \text{m}^2$$

Clave D

10. Área del sector circular:

$S = \frac{L^2}{2 . \theta}$

$$\left.\begin{array}{l}L = 12\ \text{m} \\ \theta = 3\ \text{rad}\end{array}\right\} \Rightarrow S = \frac{(12)^2}{2(3)} = 24\ \text{m}^2$$

Clave D

11. $\left.\begin{array}{l}\theta = 0{,}8\ \text{rad} \\ L = 4\ \text{m}\end{array}\right\}$ Área del sector circular:

$S = \frac{L^2}{2 . \theta}$

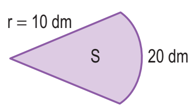
$S = \frac{(4)^2}{2(0{,}8)}$

$S = 10\ \text{m}^2$

Clave E

Resolución de problemas

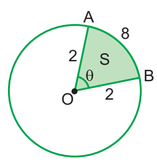
12.



$S = \frac{L . r}{2} = \frac{20.10}{2} = 100\ \text{dm}^2$

Clave B

13.



Piden: el área del sector circular (S).

$S = \frac{L.R}{2} = \frac{(8).(2)}{2} = 8$

$\therefore\ S = 8\ \text{cm}^2$

Clave E

14. $\theta = \frac{25\pi}{24}$ rad

$R = 2\sqrt{6}$ m

Piden el área del sector circular:

$S = \frac{R^2\theta}{2}$

$$S = \frac{(2\sqrt{6})^2 . \frac{25\pi}{24}}{2} = \frac{24 . 25\pi}{24 . 2}$$

$S = \frac{25\pi}{2}\ \text{m}^2$

Clave C

Nivel 2 (página 26) Unidad 2

Comunicación matemática

15. Sea S el área del círculo:

$S = \frac{1}{2}(2\pi)R^2 = \pi(6)^2 = 36\pi$

$S = 36\pi\ \text{m}^2$

I. El área de las 3/4 partes del círculo será:
$\frac{3}{4}S = \frac{3}{4}(36\pi\ \text{m}^2) = 27\pi\ \text{m}^2$

II. El área de 1/2 del círculo:
$\frac{1}{2}S = \frac{1}{2}(36\pi\ \text{m}^2) = 18\pi\ \text{m}^2$

III. El área de las 3/5 del círculo será:
$\frac{3}{5}S = \frac{3}{5}(36\pi\ \text{m}^2) = \frac{108\pi}{5} = 21{,}6\pi\ \text{m}^2$

Clave E

16. Se sabe:

$S = \frac{1}{2}\theta R^2$ ... (1)

Dato: $\theta . S = 8 \Rightarrow \theta = 8/S$ ... (2)

(2) en (1):

$S = \frac{1}{2} . \frac{8}{S} R^2$; $S^2 = 4R^2$
$S^2 = (2R)^2$

Luego:

$S = 2R \Rightarrow \frac{S}{R} = \frac{2}{1}$

$\therefore$ S es a R como 2 es a 1.

Clave C

Razonamiento y demostración

17. Área del sector circular: $S = \frac{\theta . R^2}{2}$

$$\left.\begin{array}{l}\theta = 30° = \frac{\pi}{6}\ \text{rad} \\ R = 24\ \text{m}\end{array}\right\} \Rightarrow S = \frac{\left(\frac{\pi}{6}\right)(24)^2}{2} = 48\pi$$

Clave C

18. Área del sector circular: $S = \frac{\theta . R^2}{2}$

$$\left.\begin{array}{l}\theta = 60° = \frac{\pi}{3}\ \text{rad} \\ R = 6\ \text{cm}\end{array}\right\} \Rightarrow S = \frac{\left(\frac{\pi}{3}\right)(6)^2}{2} = 6\pi$$

Clave B

19. Aplicando el área del trapecio circular:

$A = \left(\frac{L_1 + L_2}{2}\right).b$

$$\left.\begin{array}{l}L_1 = 4\ \text{m} \\ L_2 = 10\ \text{m} \\ b = 2\ \text{m}\end{array}\right\} \Rightarrow A = \left(\frac{4+10}{2}\right).2 = 14$$

Clave A

20. $\theta = 22°30' = 22° + 30' = 22° + 0{,}5° = 22{,}5°$

$\theta = 22{,}5°\left(\frac{\pi\ \text{rad}}{180°}\right) = \frac{\pi}{8}$ rad

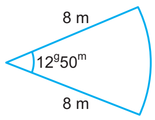
R = 12 m

Área del sector circular:

$$S = \frac{\theta . R^2}{2} = \frac{\left(\frac{\pi}{8}\right)(12)^2}{2} = 9\pi$$

Clave D

21.



$\theta = 12^g + 50^m \cdot \dfrac{1^g}{100^m} = 12{,}5^g$

$\theta = 12{,}5^g \cdot \left(\dfrac{\pi \text{ rad}}{200^g}\right) = \dfrac{\pi}{16}$ rad

El área del sector circular: $S = \dfrac{\theta . R^2}{2}$

Entonces:

$S = \dfrac{\left(\frac{\pi}{16}\right)(8)^2}{2} = 2\pi$

Clave A

Resolución de problemas

22. $\theta = 30° = \dfrac{\pi}{6}$; $L = 2\pi$ m } Área del sector circular: $S = \dfrac{L^2}{2 . \theta}$

$S = \dfrac{(2\pi)^2}{2\left(\frac{\pi}{6}\right)} = 12\pi$

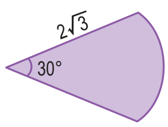
Clave A

23. $\theta = 20^g = \dfrac{\pi}{10}$ rad ; $L = \pi$ } Área del sectorcircular: $S = \dfrac{L^2}{2\theta}$

$\Rightarrow S = \dfrac{(\pi)^2}{2\left(\frac{\pi}{10}\right)} = 5\pi$

Clave E

24.



30° a radianes:

$\dfrac{S}{9} = \dfrac{20R}{\pi}$

$\dfrac{30}{9} = \dfrac{20R}{\pi}$

$R = \dfrac{\pi}{6} \Rightarrow \theta = \dfrac{\pi}{6}$ rad

$S = \dfrac{\theta . r^2}{2}$

$S = \dfrac{\pi}{6} \dfrac{(2\sqrt{3})^2}{2}$

$S = \dfrac{\pi}{6} . \dfrac{12}{2} = \pi$ m^2

Clave A

Nivel 3 (página 27) Unidad 2

Comunicación matemática

25.

I. Datos: $L = 3\pi$ m, $\theta = \pi/2$ rad

$S\triangleleft_{AOB} = \dfrac{L^2}{2\theta} = \dfrac{(3\pi)^2}{2 . \frac{\pi}{2}} = 9\pi$

$S\triangleleft_{AOB} = 9\pi$ m^2

II. Datos: $R = 2$ m, $\theta = 45°$

$45° = 45° . \dfrac{\pi \text{ rad}}{180°} = \dfrac{\pi}{4}$ rad

Luego:

$S\triangleleft_{AOB} = \dfrac{1}{2} . \theta R^2 = \dfrac{1}{2} . \dfrac{\pi}{4} . 2^2 = \dfrac{\pi}{2}$

$S\triangleleft_{AOB} = \pi/2$ m^2

III. Datos: $R = 3$ cm; $L = 6\pi$ cm

$S\triangleleft_{AOB} = \dfrac{LR}{2} = \dfrac{(6\pi)(3)}{2} = 9\pi$

$S\triangleleft_{AOB} = 9\pi$ cm^2

Clave B

26.

a. Del enunciado; "dado que S y L son equivalentes":

De la expresión:

$S = \dfrac{L . R}{2}$; pero $S = L$

$\Rightarrow L = \dfrac{L . R}{2}$; $R = 2$

Luego:

$R\theta = L \Rightarrow 2 . \theta = L$

$2\theta = L$

$\therefore$ L es igual a 2θ (verdadero).

b. De la proposición, R es igual a 3 u. De la expresión:

$S = \dfrac{L . R}{2}$; pero $R = 3$ u

$S = \dfrac{L . 3}{2} \Rightarrow L = \dfrac{2}{3}S$

$\therefore$ L es igual a 2/3 de S (verdadero).

c. Por la proposición, $\theta . S = 2$

De la expresión:

$S = \dfrac{L^2}{2\theta}$; $2\theta . S = L^2$

$2 . 2 = L^2$

$L = 2$ ... (1)

De la expresión: $S = \dfrac{L . R}{2}$; de (1):

$S = \dfrac{L . R}{2} = \dfrac{2 . R}{2}$

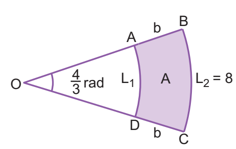
$S = R$

$\therefore$ S y R son iguales (verdadero).

Clave C

Razonamiento y demostración

27.



El área de un trapecio circular se puede calcular como:

$A = \dfrac{(L_1 + L_2)}{2} . b$

Como: $\theta = \dfrac{L_2 - L_1}{b}$

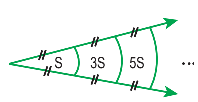
$\Rightarrow L_1 = 4$m

Para: $L_1 = 4$ m

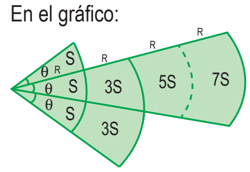
$L_2 = 8$ m ; $b = 3$ m } $\Rightarrow A = \left(\dfrac{8+4}{2}\right) . 3 = 18$

Clave C

28. Sabemos:



En el gráfico:



$\Rightarrow$ Área total = A

$21S = A$

$21\left(\dfrac{1}{2}\theta . R^2\right) = A$

$\Rightarrow \dfrac{21}{2} = \dfrac{A}{\theta . R^2}$ ... (1)

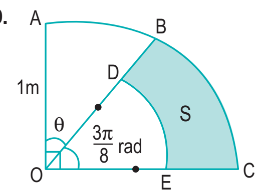
Nos Piden: $E = \dfrac{4A}{\theta . R^2}$ ... (2)

Reemplazando (1) en (2):

$E = 4\left(\dfrac{21}{2}\right) \Rightarrow E = 42$

Clave A

29.



Del gráfico:

$\theta + \dfrac{3\pi}{8} = \dfrac{\pi}{2}$

$\theta = \dfrac{\pi}{2} - \dfrac{3\pi}{8} \Rightarrow \theta = \dfrac{\pi}{8}$

Del enunciado:

$S\triangleleft_{AOB} = S\triangleleft_{DOE}$

$\dfrac{1}{2}\theta . 1^2 = S\triangleleft_{DOE} \Rightarrow S\triangleleft_{DOE} = \dfrac{\pi}{2}$ ... (1)

Además:

$S + S\triangleleft_{AOB} \Rightarrow S\triangleleft_{DOE} = S\triangleleft_{AOC}$

$S + \dfrac{\theta}{2} + \dfrac{\theta}{2} = \dfrac{1}{2}\left(\dfrac{\pi}{2}\right)1^2$

$S + \underbrace{\theta}_{\frac{\pi}{8}} = \dfrac{\pi}{4} \Rightarrow S = \dfrac{\pi}{8}$m^2

Clave D

30. Del gráfico:

$4\theta = 180°$

$\theta = 45° = 45°\left(\dfrac{\pi \text{rad}}{180°}\right) \Rightarrow \theta = \dfrac{\pi}{4}$rad

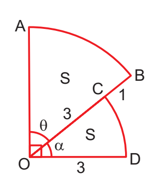
Luego:

$S = \dfrac{(3\theta)6^2}{2} = 54\theta = 54\left(\dfrac{\pi}{4}\right)$rad

$\therefore S = \dfrac{27\pi}{4}$m^2

Clave A

31.



$\Rightarrow S = \dfrac{\theta(4)^2}{2} = 8\theta$

$\Rightarrow S = \dfrac{\alpha(3)^2}{2} = \dfrac{9}{2}\alpha$

Igualando: $8\theta = \dfrac{9}{2}\alpha$

$\Rightarrow \dfrac{\theta}{\alpha} = \dfrac{9}{16}$ ...(I)

Del gráfico: $\theta + \alpha = 90° \left(\frac{10^g}{9°}\right) = 100^g$

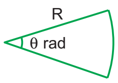
$\Rightarrow \theta + \alpha = 100^g$ ...(II)

De: (I) y (II); $\theta = 36^g \wedge \alpha = 64^g$

Clave D

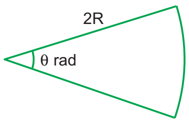
Resolución de problemas

32.



$\underbrace{\text{Área del sector} = S}$

$\frac{\theta \cdot R^2}{2} = S \Rightarrow \theta R^2 = 2S$ ...(I)



Área del sector = A

$\frac{\theta \cdot (2R)^2}{2} = A \Rightarrow A = 2\theta R^2$ ...(II)

De (I) y (II): $A = 2(2S) = 4S$

Clave C

33. Del enunciado tenemos:

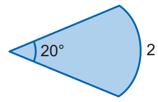
$\underbrace{\text{Área del sector circular}}_{\left(\frac{L.R}{2}\right)} = \underbrace{\text{Longitud del arco}}_{L}$

$L \cdot R = 2L$

$R = 2$

Clave E

34.



20° a radianes:

$\frac{S}{9} = \frac{20R}{\pi} \Rightarrow \frac{20}{9} = \frac{20R}{\pi}$

$R = \frac{\pi}{9} \Rightarrow \theta = \frac{\pi}{9}$ rad

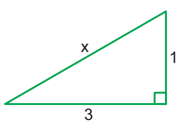
$\Rightarrow S = \frac{L^2}{2\theta} = \frac{2^2}{2\left(\frac{\pi}{9}\right)} = \frac{18}{\pi}$ cm^2

Clave E

RAZONES TRIGONOMÉTRICAS DE ÁNGULOS AGUDOS

APLICAMOS LO APRENDIDO
Nivel 1 (página 28) Unidad 2

1.



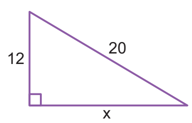
Por Pitágoras:

$x^2 = 1^2 + 3^2$

$x^2 = 1 + 9 \Rightarrow x^2 = 10 \Rightarrow x = \sqrt{10}$

Clave E

2.



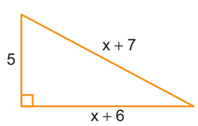
Por Pitágoras:

$20^2 = 12^2 + x^2$

$400 = 144 + x^2 \Rightarrow x^2 = 256 \Rightarrow x = 16$

Clave C

3.



Por Pitágoras:

$(x + 7)^2 = 5^2 + (x + 6)^2$

$x^2 + 14x + 49 = 25 + x^2 + 12x + 36$

$2x = 12 \Rightarrow x = 6$

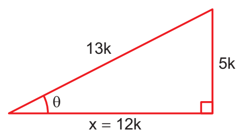
$\text{Perímetro} = 5 + (x + 6) + (x + 7)$

$\text{Perímetro} = 2x + 18 = 2(6) + 18$

$\text{Perímetro} = 30$

Clave C

4. $\text{sen}\theta = \frac{5}{13} \Rightarrow$



Por Pitágoras:

$(13k)^2 = (5k)^2 + x^2$

$12k = x$

$\Rightarrow \cos\theta = \frac{12k}{13k} = \frac{12}{13}$

Reemplazando en la expresión:

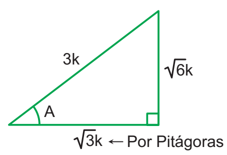
$E = 26\cos\theta + 3$

$E = 26\left(\frac{12}{13}\right) + 3 = 27$

Clave B

5.

$\text{sen}A = \frac{\sqrt{6}}{3} \Rightarrow$



$\Rightarrow \sec A = \frac{H}{CA} = \frac{3k}{\sqrt{3}\,k} = \sqrt{3}$

$\Rightarrow \tan A = \frac{CO}{CA} = \frac{\sqrt{6}\,k}{\sqrt{3}\,k} = \sqrt{2}$

Reemplazando en la expresión:

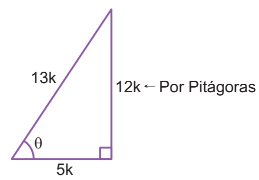
$M = 3 + \sqrt{6}\sec A - 2\tan A$

$M = 3 + \sqrt{6}\,.(\sqrt{3}) - 2(\sqrt{2})$

$M = 3 + 3\sqrt{2} - 2\sqrt{2} = 3 + \sqrt{2}$

Clave A

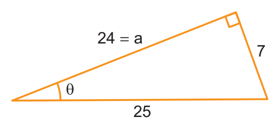
6. $\cos\theta = \frac{5}{13}$



$\Rightarrow \tan\theta = \frac{CO}{CA} = \frac{12k}{5k} = \frac{12}{5}$

Clave E

7.



Por Pitágoras:

$(25)^2 = 7^2 + a^2 \Rightarrow a = 24$

$\Rightarrow \csc\theta = \frac{25}{7} \wedge \cot\theta = \frac{24}{7}$

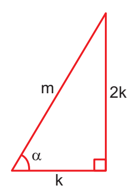
Reemplazando en la expresión:

$T = \csc\theta - \cot\theta = \frac{25}{7} - \frac{24}{7} = \frac{1}{7}$

Clave E

8.

$\tan\alpha = \frac{2}{1} \Rightarrow$



Por Pitágoras:

$m^2 = (2k)^2 + k^2$

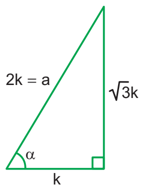
$m = \sqrt{5}\,k$

$\Rightarrow \text{sen}\alpha = \frac{CO}{H} = \frac{2k}{m} = \frac{2k}{\sqrt{5}\,k} = \frac{2}{\sqrt{5}}$

$\text{sen}^2\alpha = \frac{4}{5}$

Clave A

9. $\tan\alpha = \frac{\sqrt{3}}{1} \Rightarrow$



Por Pitágoras:

$a^2 = (\sqrt{3}\,k)^2 + (k)^2$

$a = 2k$

$\sec\alpha = \frac{2k}{k} = 2 \;\wedge\; \csc\alpha = \frac{2k}{\sqrt{3}\,k} = \frac{2}{\sqrt{3}}$

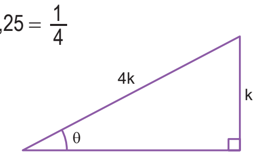
Reemplazando en la expresión:

$S = \sec^4\alpha + 6\csc^2\alpha$

$S = (2)^4 + 6(\frac{2}{\sqrt{3}})^2 = 16 + 8 = 24$

Clave D

10. $\text{sen}\theta = 0{,}25 = \frac{1}{4}$

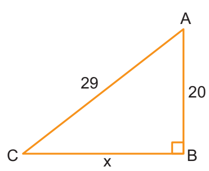


Por Pitágoras ⟶ $\sqrt{15}k$

$\Rightarrow \cot\theta = \frac{CA}{CO} = \frac{\sqrt{15}\,k}{k} = \sqrt{15}$

Clave C

11. De los datos: sea el triángulo rectángulo ABC.



Por teorema de Pitágoras:

$x^2 + 20^2 = 29^2$

$x^2 = 29^2 - 20^2$

$x^2 = (29 - 20)(29 + 20)$

$x^2 = (9)(49)$

$x = 21$

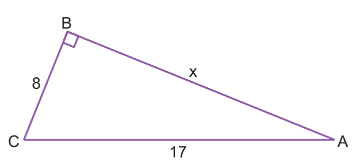
Nos piden:

Σ catetos $= AB + CB = 20 + 21$

$\therefore\ \Sigma$ catetos $= 41$

Clave C

12. Aplicando el teorema de Pitágoras en el ⊿ABC:



$17^2 = 8^2 + x^2$

$x^2 = 17^2 - 8^2$

$x^2 = (17 + 8)(17 - 8)$

$x^2 = (25)(9)$

$x = 15$

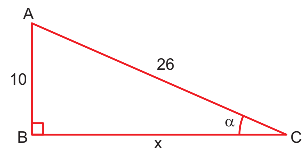
Nos piden:

$\frac{CB}{BA}$ o $\frac{BA}{CB}$: razón entre catetos

$\therefore\ \frac{CB}{BA} = \frac{8}{15}$

Clave B

13. Sea el ⊿ABC y α el ángulo de cateto opuesto 10:



Por el teorema de Pitágoras:

$10^2 + x^2 = 26^2$

$x^2 = 26^2 - 10^2$

$x^2 = 576$

$x = 24$

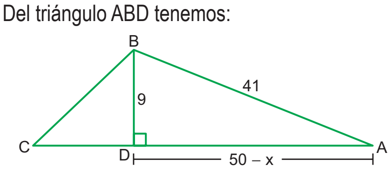
Nos piden $\cos\alpha$:

$\cos\alpha = \frac{CA}{H} = \frac{24}{26}$

$\therefore\ \cos\alpha = \frac{12}{13}$

Clave E

14. Del triángulo ABD tenemos:



Aplicando teorema Pitágoras en el ⊿ABD:

$(50 - x)^2 + 9^2 = 41^2$

$(50 - x)^2 = 41^2 - 9^2$

$(50 - x)^2 = (41 + 9)(41 - 9)$

$(50 - x)^2 = (50)(32)$

$(50 - x)^2 = (25)(64)$

$50 - x = (5)(8)$

$50 - x = 40$

$\therefore\ x = 10$

Clave C

PRACTIQUEMOS

Nivel 1 (página 30) Unidad 2

Comunicación matemática

1.

	α	θ
seno	$\frac{9}{41}$	$\frac{40}{41}$
coseno	$\frac{40}{41}$	$\frac{9}{41}$
tangente	$\frac{9}{40}$	$\frac{40}{9}$

2. Usando el cuadro de la pregunta 1:

1. $\text{sen}\theta = \frac{40}{41}$ ... (F)
2. $\cos\theta + \cos\alpha = \frac{9}{41} + \frac{40}{41} = \frac{49}{41}$ ... (F)
3. $\text{sen}\theta - \text{sen}\alpha = \frac{40}{41} - \frac{9}{41} = \frac{31}{41}$ ... (V)

Clave B

3. Usando el cuadro de la pregunta 1:

I. $\tan\alpha = \frac{9}{40}$
... Ic

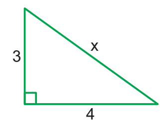
II. Complemento de θ es α, luego $\text{sen}\alpha = \frac{9}{41}$.
... IIa

III. $\tan\theta = \frac{40}{9}$
... IIIb

Clave B

Razonamiento y demostración

4.



Por Pitágoras:

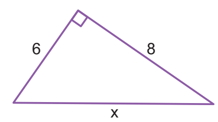
$x^2 = 3^2 + 4^2$

$x^2 = 9 + 16 = 25$

$x = \sqrt{25} = 5$

Clave E

5.



Por Pitágoras:

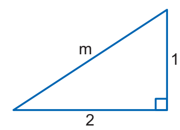
$x^2 = 6^2 + 8^2$

$x^2 = 36 + 64 = 100$

$x = \sqrt{100} = 10$

Clave D

6.



Por Pitágoras:

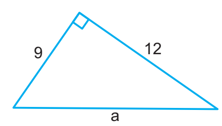
$m^2 = 1^2 + 2^2$

$m^2 = 1 + 4 = 5$

$m^2 = 5 \Rightarrow m = \sqrt{5}$

Clave C

7.



Por Pitágoras:

$a^2 = 9^2 + 12^2$

$a^2 = 81 + 144 = 225$

$a = \sqrt{225} = 15$

Clave A

8. $M = \frac{35}{12} + \frac{37}{12} = \frac{72}{12}$

$\therefore M = 6$

Clave D

9. En el ⊿ABC

Con respecto a θ:

$\frac{21}{29} = \frac{CA}{H} = \cos\theta$

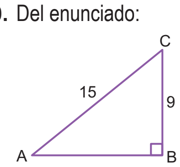
Con respecto a α:

$\frac{21}{29} = \frac{CO}{H} = \text{sen}\alpha$

Clave A

Resolución de problemas

10. Del enunciado:



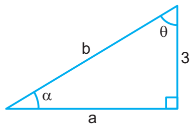
Nos piden:

$\cos C = \frac{CA}{H} = \frac{9}{15}$

$\therefore \cos C = \frac{3}{5}$

Clave D

11. Sea α el ángulo de cateto opuesto igual a 3.



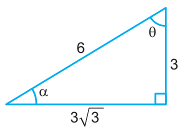
Por dato:

$3^2 + a^2 = 36 \Rightarrow a = 3\sqrt{3}$

Del gráfico:

$3^2 + a^2 = b^2 = 36 \Rightarrow b = 6$

El triángulo queda:

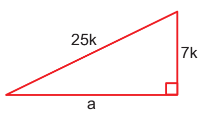


Nos piden $\tan\theta$:

$\tan\theta = \frac{3\sqrt{3}}{3} = \sqrt{3}$

Clave C

12. De los datos:



Por teorema de Pitágoras:

$a^2 + (7k)^2 = (25k)^2$

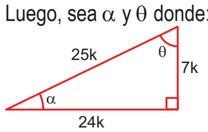
$a^2 = (25k)^2 - (7k)^2$

$a^2 = (25k + 7k)(25k - 7k)$

$a^2 = (32k)(18k)$

$a = 24k$

Luego, sea α y θ donde:



$\text{sen}\theta = \frac{24k}{25k} = \frac{24}{25}$

$\text{sen}\alpha = \frac{7k}{25k} = \frac{7}{25}$

Nos piden el mayor de los senos:

$\therefore \text{sen}\theta = \frac{24}{25}$

Clave D

Nivel 2 (página 31) Unidad 2

13. Por las definiciones de las razones trigonométricas, completemos el cuadro.

	RT	Definición
1	d	$\frac{H}{CA}$
2	c	$\frac{CA}{CO}$
3	a	$\frac{CO}{H}$
4	b	$\frac{H}{CO}$

Clave E

14. Por las definiciones de RT:

I. $\text{sen}\theta = \frac{CO}{H}$

II. $\sec\theta = \frac{H}{CA}$

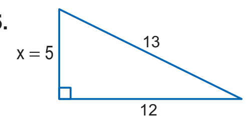
III. $\cot\theta = \frac{CA}{CO}$

$\therefore$ Ia IIb IIIc

Clave E

Razonamiento y demostración

15.



Por Pitágoras:

$13^2 = x^2 + 12^2$

$169 = x^2 + 144$

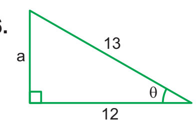
$25 = x^2$

$\Rightarrow x = 5$

$\therefore$ El perímetro será: $13 + 5 + 12 = 30$

Clave D

16.



Por Pitágoras:

$13^2 = a^2 + 12^2 \Rightarrow a = 5$

$\Rightarrow \cot\theta = \frac{12}{a} = \frac{12}{5}$

$\Rightarrow \csc\theta = \frac{13}{a} = \frac{13}{5}$

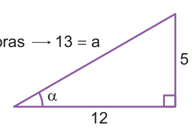
Reemplazando en la expresión:

$E = \cot\theta + \csc\theta$

$E = \frac{12}{5} + \frac{13}{5} = \frac{25}{5} = 5$

Clave C

17. Por Pitágoras $\longrightarrow 13 = a$



$\Rightarrow \sec\alpha = \frac{H}{CA} = \frac{13}{12}$

$\Rightarrow \tan\alpha = \frac{CO}{CA} = \frac{5}{12}$

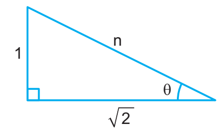
Reemplazando en la expresión:

$S = \sec\alpha - \tan\alpha$

$S = \left(\frac{13}{12}\right) - \left(\frac{5}{12}\right) = \frac{8}{12} = \frac{2}{3}$

Clave B

18.



Por Pitágoras:

$n^2 = 1^2 + (\sqrt{2})^2 = 3$

$\Rightarrow n = \sqrt{3}$

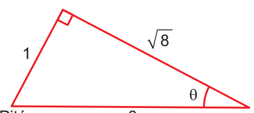
$\Rightarrow \sec\theta = \frac{H}{CA} = \frac{n}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$

En la expresión:

$E = \sec^2\theta = \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2 = \frac{3}{2}$

Clave B

19.



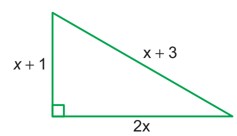
Por Pitágoras $\longrightarrow a = 3$

$\Rightarrow \cos\theta = \frac{CA}{H} = \frac{\sqrt{8}}{3}$

$\Rightarrow 18\cos^2\theta = 18\left(\frac{8}{9}\right) = 16$

Clave A

20.



Por Pitágoras:

$(x+3)^2 = (x+1)^2 + (2x)^2$

$x^2 + 6x + 9 = x^2 + 2x + 1 + 4x^2$

$0 = 4x^2 - 4x - 8$

$\Rightarrow x^2 - x - 2 = 0$

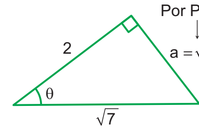
$(x-2)(x+1) = 0$

$\Rightarrow x = 2 \lor x = -1$

$\Rightarrow$ Del gráfico, x es positivo. $\therefore x = 2$

Clave D

21.

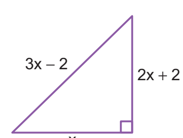


Piden: $E = \cos^2\theta + \text{sen}^2\theta$

$E = \left(\frac{2}{\sqrt{7}}\right)^2 + \left(\frac{\sqrt{3}}{\sqrt{7}}\right)^2 = \frac{4}{7} + \frac{3}{7} = \frac{7}{7} = 1$

Clave C

22.



Por Pitágoras:
$(3x-2)^2=(2x+2)^2+x^2$
$9x^2-12x+4=4x^2+8x+4+x^2$
$4x^2-20x=0$
$4x(x-5)=0$
$\Rightarrow x=0 \quad \vee \quad x=5$
Del gráfico, x no puede ser cero.
Entonces: x = 5

ClaveE

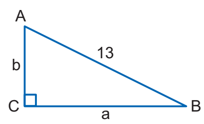
Resolución de problemas

23. Por Pitágoras:

$(2)^2+(\sqrt{5})^2=(x+1)^2$
$9=(x+1)^2$
$3=x+1$
$\Rightarrow x=2$

Clave D

24. Por dato:



$\tan A=\dfrac{a}{b}=\dfrac{12}{5} \Rightarrow \begin{matrix} a=12k \\ b=5k \end{matrix}$

Por el teorema de Pitágoras:
$a^2+b^2=13^2$
$(12k)^2+(5k)^2=13^2$
$144k^2+25k^2=169$
$169k^2=169$
$k^2=1$
$k=1$

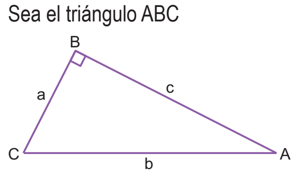
Entonces
a = 12 ; b = 5
Nos piden:

$\cos B=\dfrac{a}{13}=\dfrac{12}{13}$

$\therefore \cos B=\dfrac{12}{13}$

Clave A

25. Sea el triángulo ABC



Por dato
$\dfrac{b-a}{b+a}=\dfrac{2}{3}$
$3b-3a=2b+2a$
$b=5a$
$\dfrac{a}{b}=\dfrac{1}{5}$ ... (1)

Nos piden:
$\text{sen}A=\dfrac{a}{b}$

De (1):
$\therefore \text{sen}A=\dfrac{1}{5}$

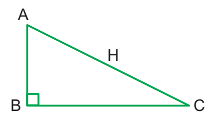
Clave A

Nivel 3 (página 32) Unidad 2

Comunicación matemática

26.

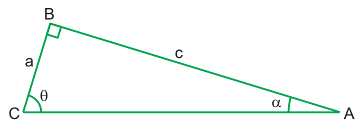
1. Por teorema de correspondencia en un triángulo. A mayor longitud del lado en un triángulo se le opone un mayor ángulo.



En el ◺ABC, el mayor ángulo es el ángulo recto. Por lo tanto, el lado que se le opone (hipotenusa) es el mayor de los lados ...(V)

2. El teorema de Pitágoras se cumple solo en los triángulos rectángulos. ... (F)

3. Sea el triángulo rectángulo ABC:



Sea α el menor ángulo agudo y r la razón entre catetos, es decir:

$r=\dfrac{a}{c}$ o $r=\dfrac{c}{a}$

Por teorema de correspondencia:

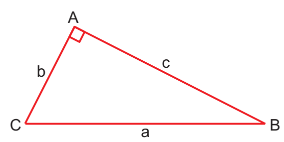
$a<c \Rightarrow \dfrac{a}{c}<1$

Se concluye

Si $r=\dfrac{a}{c}=\tan\alpha \Rightarrow r<1$ ... (V)

Clave E

27. En el ◺ ABC las longitudes de los lados correspondientes a cada ángulo se representan con letras minúsculas según corresponda. Del dato ◺recto en A.



I. c representa la longitud del lado opuesto al ángulo C.

II. a es la representación de la longitud del mayor lado en el ◺ABC (hipotenusa).

III. El cateto opuesto al ángulo B se representa con la letra b minúscula.

Clave B

Razonamiento y demostración

28. $\tan\theta=4=\dfrac{CB}{AB}$

$4=\dfrac{CB}{2BP} \Rightarrow 8=\dfrac{CB}{BP}$ ...(1)

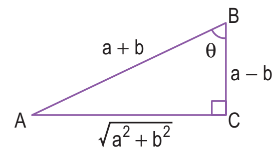
Nos Piden:

$\tan\alpha=\dfrac{BP}{CB}=\dfrac{1}{\dfrac{CB}{BP}}$ ...(2)

Reemplazando (1) en (2):
$\tan\alpha=\dfrac{1}{8}$

Clave A

29.



Del gráfico, el mayor cateto es: $\sqrt{a^2+b^2}$

Nos piden entonces: $\cos\theta=\dfrac{a-b}{a+b}$...(1)

Además, por el teorema de Pitágoras:
$(a+b)^2=(a-b)^2+(\sqrt{a^2+b^2})^2$

Reduciendo terminos tenemos:
$4ab=a^2+b^2$ ...(2)

Elevando (1) al cuadrado:
$\cos^2\theta=\dfrac{a^2+b^2-2ab}{a^2+b^2+2ab}$...(3)

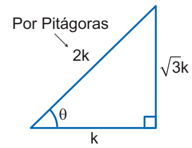
Reemplazando (2) en (3):
$\cos^2\theta=\dfrac{4ab-2ab}{4ab+2ab}=\dfrac{2ab}{6ab}=\dfrac{1}{3}$

$\therefore \cos\theta=\dfrac{1}{\sqrt{3}}$

Clave D

30.

$\cot\theta=\dfrac{1}{\sqrt{3}} \Rightarrow$

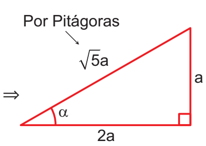


$\Rightarrow \text{sen}\theta=\dfrac{CO}{H}=\dfrac{\sqrt{3}\,k}{2k}=\dfrac{\sqrt{3}}{2}$

Clave D

31.

$\tan\alpha=0{,}5=\dfrac{1}{2} \Rightarrow$

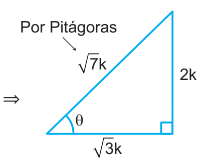


$\Rightarrow \cos\alpha=\dfrac{CA}{H}=\dfrac{2a}{\sqrt{5}\,a}=\dfrac{2}{\sqrt{5}}$

Clave D

32.

$\tan\theta=\dfrac{2}{\sqrt{3}} \Rightarrow$

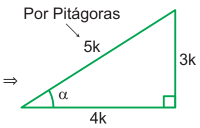


$\Rightarrow \text{sen}\theta=\dfrac{CO}{H}=\dfrac{2k}{\sqrt{7}\,k}=\dfrac{2}{\sqrt{7}}$

Clave C

33.

$\tan\alpha = 0{,}75 = \frac{3}{4} \Rightarrow$



$\Rightarrow \csc\alpha = \frac{H}{CO} = \frac{5k}{3k} = \frac{5}{3}$

$\Rightarrow \cot\alpha = \frac{CA}{CO} = \frac{4k}{3k} = \frac{4}{3}$

Reemplazando en la expresión:

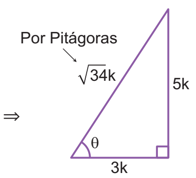
$E = \csc\alpha - \cot\alpha$

$E = \left(\frac{5}{3}\right) - \left(\frac{4}{3}\right) = \frac{1}{3}$

Clave C

34.

$\cot\theta = \frac{3}{5} \Rightarrow$



$\Rightarrow \sec\theta = \frac{H}{CA} = \frac{\sqrt{34}\,k}{3k} = \frac{\sqrt{34}}{3}$

$\Rightarrow \tan\theta = \frac{CO}{CA} = \frac{5k}{3k} = \frac{5}{3}$

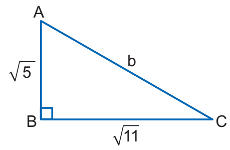
Reemplazando en la expresión:

$M = \left(\frac{\sqrt{34}}{3}\right) - \left(\frac{5}{3}\right) = \frac{\sqrt{34} - 5}{3}$

Clave B

Resolución de problemas

35. De los datos, construimos el triángulo rectángulo ABC (recto en B):



Por teorema de correspondencia:

$AB < BC \Rightarrow m\angle A > m\angle C$

A : ángulo agudo mayor

Por T. Pitágoras:

$(\sqrt{5})^2 + (\sqrt{11})^2 = b^2$

$5 + 11 = b^2$

$16 = b^2$

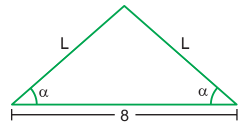
$b = 4$

Nos piden:

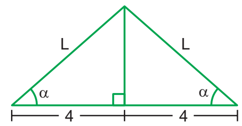
$\cos A = \frac{\sqrt{5}}{b} \qquad \therefore \cos A = \frac{\sqrt{5}}{4}$

Clave B

36. Del enunciado, sea a los ángulos iguales.



Trazamos altura con respecto a la base:



Por dato: $\cos\alpha = \frac{2}{7}$

Del triángulo:

$\cos\alpha = \frac{4}{L} = \frac{2}{7}$

$\frac{2}{L} = \frac{1}{7}$

$\therefore L = 14$

Clave C

PROPIEDADES DE LAS RAZONES TRIGONOMÉTRICAS

APLICAMOS LO APRENDIDO (página 33) Unidad 2

1. $\text{sen}6x = \cos 4x \quad \Rightarrow \quad 6x + 4x = 90°$

$10x = 90°$

$x = 9°$

Clave B

2.

$\tan 3x = \cot 7x \quad \Rightarrow \quad 3x + 7x = 90°$

$10x = 90°$

$x = 9°$

Clave E

3.

$\tan(2\alpha + 2x) = \cot(3x - 2\alpha)$

$\Rightarrow (2\alpha + 2x) + (3x - 2\alpha) = 90°$

$2\alpha + 2x + 3x - 2\alpha = 90°$

$5x = 90°$

$x = 18°$

Clave A

4. $\text{sen}(\alpha + \theta) = \cos(8\alpha - \theta)$

$\Rightarrow (\alpha + \theta) + (8\alpha - \theta) = 90°$

$9\alpha = 90°$

$\alpha = 10°$

Clave C

5. $\cot(3x - 60°) = \tan(x + 50°)$

$\Rightarrow (3x - 60°) + (x + 50°) = 90°$

$4x - 10° = 90°$

$4x = 100°$

$x = 25°$

Clave D

6. $\cos(x + 8°) = \text{sen}(x + 2°)$

$\Rightarrow (x + 8°) + (x + 2°) = 90°$

$2x + 10° = 90°$

$2x = 80°$

$x = 40°$

Clave B

7. $\tan(x - 24°)\cot(60° - x) = 1$

$\Rightarrow x - 24° = 60° - x$

$2x = 84°$

$x = 42°$

Clave D

8. $E = \left[\frac{5\tan 3°}{\cot 87°} - \frac{2\sec 28°}{\csc 62°}\right]^2$

Por ser ángulos complementarios:

$\tan 3° = \cot 87°$

$\sec 28° = \csc 62°$

Reemplazando:

$E = \left[\frac{5\tan 3°}{\tan 3°} - \frac{2\sec 28°}{\sec 28°}\right]^2 = (5 - 2)^2$

$E = 3^2 = 9$

Clave A

9. $\text{sen}4x \,.\, \csc(x + 30°) = 1$

$\Rightarrow 4x = x + 30°$

$3x = 30°$

$x = 10°$

Clave E

10. $\cos(3x + 1°) \,.\, \sec(5x - 49°) = 1$

$\Rightarrow (3x + 1°) = (5x - 49°)$

$49° + 1° = 5x - 3x$

$50° = 2x$

$x = 25°$

Clave B

11. Del dato:

$\text{sen}30° = \cos(4x) \Rightarrow 30° + 4x = 90°$

$4x = 60°$

$x = 15°$

Nos piden:

$3x = 3(15°) = 45°$

Transformando a radianes:

$45° = 45°.\ \frac{\pi}{180°}\text{rad} = \frac{\pi}{4}\text{rad}$

$\therefore 3x = \frac{\pi}{4}\text{rad}$

Clave C

12. Dato:

$\tan\left(\frac{\pi}{4}+3x\right).\cot\left(\frac{\pi}{6}+4x\right) = 1$

$\Rightarrow \frac{\pi}{4} + 3x = \frac{\pi}{6} + 4x$

$x = \frac{\pi}{4} - \frac{\pi}{6}$

$x = \frac{\pi}{12}$ rad

Nos piden 2x:

$\therefore\ 2x = \frac{\pi}{6}$ rad

Clave D

13. $M = 3\cos66°\csc24° + 1$

Para ángulos complementarios:

$\csc24° = \sec66°$

$\Rightarrow\ M = 3\underbrace{\cos66°\sec66°}_{1} + 1$

$M = 3(1) + 1$

$\therefore\ M = 4$

Clave E

14. De la expresión:

$\cos\left(2x+\frac{\pi}{6}\right)\sec\left(\frac{\pi}{2}-x\right) = 1$

$\Rightarrow\ 2x + \frac{\pi}{6} = \frac{\pi}{2} - x$

$3x = \frac{\pi}{2} - \frac{\pi}{6}$

$x = \frac{\pi}{9}$ rad

En el sistema sexagesimal:

$x = \frac{\pi}{9}\text{rad} \cdot \frac{180°}{\pi\ \text{rad}}$

$\therefore x = 20°$

Clave B

PRACTIQUEMOS

Nivel 1 (página 35) Unidad 2

Comunicación matemática

1. La definición corresponde a razones "trigonométricas recíprocas o inversas".

Clave C

2. Si α y θ son ángulos complementarios se cumple:

A) $\text{sen}\theta = \cos\alpha$ ...correcto

B) $\tan\theta = \cot\alpha$ ...correcto

C) $\cos\alpha\sec(90° - \theta) = 1$

$\alpha + \theta = 90° \Rightarrow \alpha = 90° - \theta$

Luego:

$\cos\alpha\sec\alpha = 1$ ... ...correcto

(Razones recíprocas)

D) $\sec\alpha = \csc\theta.$ ...incorrecto

Clave D

Razonamiento y Demostración

3. $\text{sen}x\ .\ \csc50° = 1$

Si: $\text{sen}\alpha\ .\ \csc\beta = 1$

$\Rightarrow \alpha = \beta$

En el problema:

$x = 50°$

Clave D

4. $\sec3\alpha = \csc2\alpha$

$\Rightarrow (3\alpha) + (2\alpha) = 90°$

$5\alpha = 90°$

$\alpha = 18°$

Clave D

5. $\text{sen}x = \cos\frac{\pi}{5}$

$\Rightarrow (x) + \left(\frac{\pi}{5}\right) = \underline{90°}$

$x + \frac{\pi}{5} = \frac{\pi}{2}$

$x = \frac{\pi}{2} - \frac{\pi}{5} = \frac{5\pi - 2\pi}{10} = \frac{3\pi}{10}$

Clave E

6. $\cos y = \text{sen}8°$

$\Rightarrow y + 8° = 90°$

$y = 82°$

Clave A

7. $\cot2x = \tan40°$

$\Rightarrow (2x) + 40° = 90°$

$2x = 50°$

$x = 25°$

Clave E

8. $\csc(\alpha + 30°) = \sec48°$

$\Rightarrow (\alpha + 30°) + 48° = 90°$

$\alpha + 78° = 90°$

$\alpha = 12°$

Clave B

9. $E = (\text{sen}10°\ .\ \csc10°)^2$

Sabemos: $\text{sen}\theta\ .\ \csc\theta = 1$

$\Rightarrow E = (1)^2 = 1$

Clave C

10. $\text{sen}40° = \cos2y$

$40 + 2y = 90°$

$2y = 50°$

$y = 25°$

Clave D

11. $\text{sen}3x = \cos x$

$3x + x = 90$

$4x = 90$

$x = \frac{45}{2}$

$x = \frac{45}{2}\left(\frac{\pi}{180}\text{rad}\right) = \frac{\pi}{8}$ rad

Clave C

12. $\text{sen}\theta = \cos\theta$

$\theta + \theta = 90°$

$2\theta = 90°$

$\theta = 45°$

Clave E

13. $\text{sen}4x = \cos10°$

$4x + 10° = 90°$

$4x = 80°$

$\Rightarrow x = 20°$

Clave C

14. $\tan3x = \cot2x$

$3x + 2x = 90°$

$5x = 90°$

$\Rightarrow x = 18°$

Clave A

Resolución de problemas

15. Por dato.

$\sec\alpha = 3$

De las razones trigonométricas recíprocas:

$\cos\alpha\sec\alpha = 1$

$\cos\alpha.3 = 1$

$\therefore\ \cos\alpha = \frac{1}{3}$

Clave A

16. Sea θ el complemento de α, es decir:

$\alpha + \theta = 90°$

Por dato:

$\cot\theta = \frac{2}{5}$

De razones trigonométricas de ángulos complementarios:

$\tan\alpha = \cot\theta$

$\therefore\ \tan\alpha = \frac{2}{5}$

Clave D

17. Sea α y θ dos ángulos agudos.

Donde: $\alpha + \theta = \frac{\pi}{2}$ rad

Entonces, se cumple:

$\sec\alpha = \csc\theta$

$\therefore\ \frac{\sec\alpha}{\csc\theta} = 1$

Clave E

Nivel 2 (página 36) Unidad 2

Comunicación matemática

18.

I. $\text{sen}\alpha = \cos\theta\ \Rightarrow\ \alpha + \beta = 90°$

$\therefore\ \alpha$ y θ son complementarios (b)

II. $\tan\theta.\tan\phi = 1$

$\tan\theta\cot(90° - \phi) = 1$

$\Rightarrow\ \theta = 90° - \phi$

$\theta + \phi = 90°$

$\therefore\ \theta$ y ϕ son complementarios (b)

III. $\tan\omega\cot\beta = 1\ \Rightarrow\ \omega = \beta$

$\therefore\ \omega$ y β son iguales (a)

Clave B

19.

I. Del enunciado; sea α y β dichos ángulos; luego:

$\tan\alpha\tan\beta = 1$ ... (1)

Por ángulos complementarios:

$\tan\beta = \cot(90° - \beta)$

En (1):
$\tan\alpha\cot(90° - \beta) = 1$
$\Rightarrow \alpha = 90° - \beta$
$\alpha + \beta = 90°$
$\therefore$ α y β son complementarios. (correcta)

II. Sea θ y ω los ángulos del enunciado:
$\sec\theta = \csc\omega$
Por razones trigonométricas de ángulos complementarios:
$\theta + \omega = 90°$
$\therefore$ θ y ω son complementarios. (Incorrecta)

III. Sean β y α los ángulos mencionadas entonces:
$\beta + \alpha = 90°$
Se cumple:
$\operatorname{sen}\beta = \cos\alpha$
$\therefore \dfrac{\operatorname{sen}\beta}{\cos\alpha} = 1$ (correcta)

Clave A

Razonamiento y Demostración

20. $\cos x \,.\, \sec 30° - 1 = 0$

Si: $\cos\alpha \,.\, \sec\beta = 1$
$\Rightarrow \alpha = \beta$
En el problema:
$\cos x \,.\, \sec 30° = 1$
$\Rightarrow x = 30°$

Clave E

21. $\tan x \,.\, \cot 20° - 1 = 0 \Rightarrow \tan x \,.\, \cot 20° = 1$

Si: $\tan\alpha \,.\, \cot\beta = 1$
$\Rightarrow \alpha = \beta$
En el problema:
$x = 20°$

Clave E

22. $M = \sqrt{\tan 18° . \cot 18° + 3}$

Sabemos: $\tan\beta \,.\, \cot\beta = 1$

$\Rightarrow \quad M = \sqrt{1+3} = \sqrt{4} = 2$

Clave A

23. $\tan(2x - 14°)\tan 24° = 1$
$\tan(2x - 14°)\cot 66° = 1$
Se debe cumplir:
$2x - 14° = 66° \Rightarrow x = 40°$

Clave D

24. De la expresión:
$\operatorname{sen}2a = \cos(90° - 4b)$
De razones trigonométricas de ángulos complementarios:
$2a + 90° - 4b = 90°$
$2a = 4b$
$a = 2b$
$\therefore \dfrac{a}{b} = 2$

Clave A

Resolución de problemas

25. Sea θ el ángulo agudo, nos piden:
$3\csc\alpha.\cos(90° - \alpha)$

Luego:
$3\csc\alpha.\cos(90° - \alpha) = 3\underbrace{\csc\alpha.\operatorname{sen}\alpha}_{1}$

$\therefore$ $3\csc\alpha\cos(90° - \alpha) = 3$

Clave B

26. Por datos:
$\alpha + \theta = 45°$
$2\alpha + 2\theta = 90°$
2α y 2θ son ángulos complementarios.
Entonces:
$\sec 2\alpha = \csc 2\theta$
$\therefore \dfrac{\sec 2\alpha}{\csc 2\theta} = 1$

Clave B

27. Por dato:
$\beta + \theta = 180°$
$\dfrac{\beta}{2} + \dfrac{\theta}{2} = 90°$
$\dfrac{\beta}{2}$ y $\dfrac{\theta}{2}$ son ángulos complementarios.

Luego: $\tan\dfrac{\beta}{2} = \cot\dfrac{\theta}{2}$

$\therefore \dfrac{\tan\dfrac{\beta}{2}}{\cot\dfrac{\theta}{2}} = 1$

Clave D

Nivel 3 (página 36) Unidad 2

Comunicación matemática

28.

I. Para un ángulo el producto de dos de sus razones trigonométricas recíprocas es igual a la unidad.
... incorrecta

II. Dos ángulos complementarios suman 90°.
... incorrecta

III. Para un ángulo agudo el coseno de su complemento es igual al seno de dicho ángulo
... incorrecta

Clave E

29. α y β son complementarios entonces:

A) $\csc\alpha\operatorname{sen}(90° - \beta)$
$\alpha + \beta = 90°$
$\alpha = 90° - \beta$
$\Rightarrow \csc\alpha\operatorname{sen}\alpha = 1$
... Correcta

B) $\tan\beta = \cot\alpha$
... Correcta

C) $\sec\beta = \csc\alpha \neq \csc(90° - \alpha)$
... Incorrecta

D) $\tan\beta\tan\alpha = \tan\beta\cot(90° - \alpha)$
$\tan\beta\tan\alpha = \tan\beta\cot\beta$
$\therefore$ $\tan\beta\tan\alpha = 1$
... Correcta

Clave C

Razonamiento y demostración

30. $\cos 2x \,.\, \sec(30° - x) = 1$
$\Rightarrow (2x) = (30° - x)$
$3x = 30°$
$x = 10°$

Clave C

31. $\tan(x - 5°) \,.\, \cot(55° - x) = 1$
$\Rightarrow (x - 5°) = (55° - x)$
$2x = 60°$
$x = 30°$

Clave B

32. $\operatorname{sen}(x + 10°) = \cos(2x - 10°)$
$\Rightarrow (x + 10°) + (2x - 10°) = 90°$
$3x = 90°$
$x = 30°$

Clave D

33. $\tan(3x - 20°) = \cot(2x + 30°)$
$\Rightarrow (3x - 20°) + (2x + 30°) = 90°$
$5x + 10° = 90°$
$5x = 80° \Rightarrow x = 16°$

Clave E

34. $\sec(x + 20°) = \csc(x + 10°)$
$\Rightarrow (x + 20°) + (x + 10°) = 90°$
$2x + 30° = 90°$
$2x = 60° \Rightarrow x = 30°$

Clave C

35. $\operatorname{sen}(3x + 10°) \,.\, \csc(x + 40°) = 1$
$\Rightarrow (3x + 10°) = (x + 40°)$
$2x = 30°$
$x = 15°$

Clave A

36. $\cos(6x - 10°) \,.\, \sec(3x + 80°) = 1$
$\Rightarrow (6x - 10°) = (3x + 80°)$
$3x = 90°$
$x = 30°$

Clave A

37. $\tan 2\theta \,.\, \cot\left(\dfrac{\pi}{5} - \theta\right) = 1$
$\Rightarrow \quad (2\theta) = \left(\dfrac{\pi}{5} - \theta\right)$
$3\theta = \dfrac{\pi}{5}$
$\theta = \dfrac{\pi}{15}$

Clave C

38.
$E = \underbrace{\tan 18° . \cot 18°}_{1} + \underbrace{\cos 14° . \sec 14°}_{1} + \underbrace{\csc 32° . \operatorname{sen} 32°}_{1}$
$E = 1 + 1 + 1$
$\therefore E = 3$

Clave E

39. $\tan(8x - 8°) = \cot(x + 8°)$

$\Rightarrow (8x - 8) + (x + 8°) = 90°$

$9x = 90°$

$x = 10°$

Clave E

Resolución de problemas

40. Sea α el ángulo mencionado; nos piden.

$\csc\alpha.\underbrace{\cos(90° - \alpha)}_{\text{complemento}}$

Luego:

$\csc\alpha\cos(90° - \alpha) = \underbrace{\csc\alpha\,\text{sen}\alpha}_{1}$

$\therefore\ \csc\alpha\cos(90° - \alpha) = 1$

Clave C

41. Del enunciado, sean los ángulos α y β:

$\text{sen}\alpha = \cos\beta$

α y β son ángulos complementarios:

$\alpha + \beta = 90°$

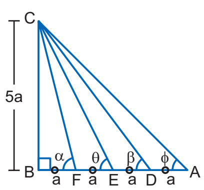
$\dfrac{\alpha + \beta}{2} = 45° = 45° \,.\, \dfrac{\pi}{180°}\,\text{rad}$

$\therefore\ \dfrac{\alpha + \beta}{2} = \dfrac{\pi}{4}\,\text{rad}$

Clave E

MARATÓN MATEMÁTICA (página 38)

1.



$5AD = CB$

$\rightarrow AD = a \Rightarrow CB = 5a$

$$M = \frac{\cot\theta + \cot\alpha}{\cot\beta + \cot\phi} = \frac{\frac{2a}{5a} + \frac{a}{5a}}{\frac{3a}{5a} + \frac{4a}{5a}} = \frac{\frac{3a}{5a}}{\frac{7a}{5a}}$$

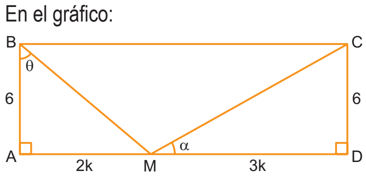
$M = \dfrac{3a}{7a} = \dfrac{3}{7}$

Clave D

2. Por dato:

$AM = 2k \ \wedge\ MD = 3k$

En el gráfico:



$\tan\theta + \cot\alpha = 5$

$\dfrac{2k}{6} + \dfrac{3k}{6} = 5$

$\dfrac{5k}{6} = 5 \Rightarrow k = 6$

$\Rightarrow BM = 6\sqrt{5} \ \wedge MC = 6\sqrt{10}$

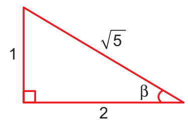
Nos piden calcular:

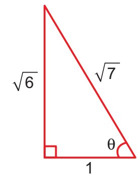
$\sec\alpha + \csc\theta = \dfrac{6\sqrt{10}}{18} + \dfrac{6\sqrt{5}}{12} = \dfrac{\sqrt{10}}{3} + \dfrac{\sqrt{5}}{2}$

$= \sqrt{5}\left(\dfrac{\sqrt{2}}{3} + \dfrac{1}{2}\right)$

Clave C

3. $\csc\beta = \sqrt{5}$ y $\sec\theta = \sqrt{7}$





Nos piden calcular:

$J = \sqrt{42}\,.\,\csc\theta + \sqrt{5}\,\cos\beta$

$J = \sqrt{42}\,.\,\dfrac{\sqrt{7}}{\sqrt{6}} + \sqrt{5}\,.\,\dfrac{2}{\sqrt{5}}$

$J = 7 + 2 = 9$

Clave B

4. $\dfrac{5\text{sen}(x + 15)°.\ \text{sen}67°}{\sec10°.\ \cos23°} = \dfrac{6\cos60°.\ \tan32°}{\csc80°.\text{ctg}58°}$

Aplicamos las propiedades de las razones trigonométricas:

$\text{sen}67° = \cos23°$

$\sec10° = \csc80°$

$\tan32° = \cot58°$

Luego:

$5\text{sen}(x + 15°) = 6\cos60°$

$\text{sen}(x + 15)° = \dfrac{6}{5}\cdot\dfrac{1}{2}$

$\text{sen}(x + 15)° = \dfrac{3}{5}$

$\Rightarrow (x + 15)° = 37°$

$x = 22$

Clave C

5. Del gráfico:

$\alpha + \theta = 90° \ \wedge\ \alpha = \beta$

$\dfrac{\text{sen}(2x + 3)°.\cos(90 - \theta)}{\cos\alpha.\ \sec\alpha} = \dfrac{\tan(90 - \alpha).\cos(3x + 17)}{\cot\beta\,.\,\csc\theta}$

$\text{sen}(2x + 3)°\,.\,\dfrac{\cancel{\text{sen}\theta}}{\cos\alpha\,.\,\sec\alpha} = \dfrac{\cot\alpha\,.\,\cos(3x + 17)}{\cot g\,.\,\dfrac{1}{\cancel{\text{sen}\theta}}}$

$\text{sen}(2x + 3)° = \cos(3x + 17)°$

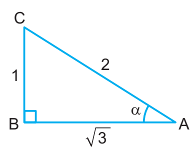
$2x + 3 + 3x + 17 = 90°$

$5x + 20 = 90°$

$5x = 70° \quad \Rightarrow \quad x = 14°$

Clave E

6. $\csc\alpha = 2$



$\Rightarrow AB = \sqrt{3}$

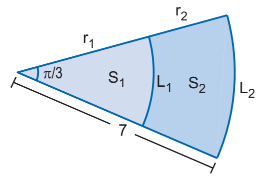
$R = [8\text{sen}\alpha + \sqrt{3}\ \sec\alpha]\csc\alpha$

$R = \left[8\,.\,\dfrac{1}{2} + \sqrt{3}\,.\,\dfrac{2}{\sqrt{3}}\right].\,2$

$R = [4 + 2]\,.\,2 \qquad \therefore R = 12$

Clave E

7.



Calculamos r_1:

$S_1 = \theta\,.\,\dfrac{r_1^2}{2}$

$\dfrac{8\pi}{3} = \dfrac{\pi}{3}\cdot\dfrac{r_1^2}{2}$

$r_1 = 4$

$\Rightarrow r_2 = 7 - 4 = 3$

Calculamos L_1:

$S_1 = \dfrac{L_1\,.\,r_1}{2}$

$\dfrac{8\pi}{3} = \dfrac{L_1\,.\,4}{2}$

$\dfrac{4\pi}{3} = L_1$

Por último calculamos L_2:

$L_2 = \dfrac{\pi}{3}\cdot 7 = \dfrac{7\pi}{3}$

Luego:

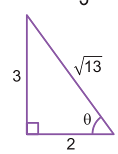
$S_2 = \left(\dfrac{L_1 + L_2}{2}\right).\,3$

$S_2 = \left(\dfrac{4\pi}{3} + \dfrac{7\pi}{3}\right)\cdot\dfrac{3}{2} = \dfrac{11\pi}{3}\cdot\dfrac{3}{2}$

$\Rightarrow\ S_2 = \dfrac{11\pi}{2}$

Clave C

8. $\cot\theta = \dfrac{2}{3}$



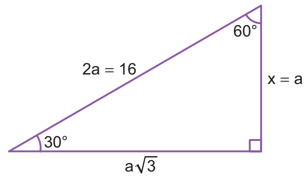
Reemplazando:

$$J = \frac{\frac{2}{\sqrt{13}} + \frac{\sqrt{13}}{2}}{\frac{\sqrt{13}}{3} + \frac{3}{\sqrt{13}}} = \frac{\frac{4 + 13}{2\sqrt{13}}}{\frac{13 + 9}{3\sqrt{13}}} = \frac{17\,.\,3}{2\,.\,22}$$

$J = \dfrac{51}{44}$

APLICAMOS LO APRENDIDO (página 40) Unidad 3

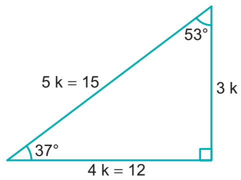
1.



Del gráfico:
$2a = 16$
$a = 8 \qquad \Rightarrow x = 8$

Clave D

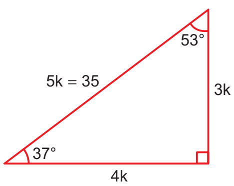
2.



Del gráfico:
$4k = 12$
$k = 3$
$\Rightarrow x = 5k = 5(3) = 15$

Clave B

3.

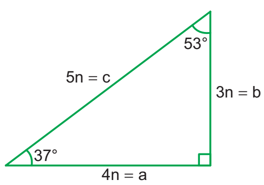


Del gráfico:
$5k = 35 \Rightarrow k = 7$

$\Rightarrow a = 4k = 4(7) = 28$
$\Rightarrow b = 3k = 3(7) = 21$
$\therefore (a + b) = 28 + 21 = 49$

Clave D

4.

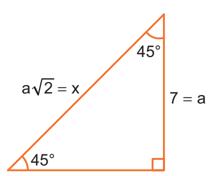


Piden: $\left(\dfrac{a+c}{b}\right)$

$\left(\dfrac{4n+5n}{3n}\right) = \dfrac{9n}{3n} = 3$

Clave D

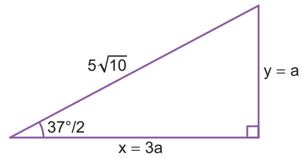
5.



Del gráfico: $a = 7$
$x = a\sqrt{2} \Rightarrow x = 7\sqrt{2}$

Clave A

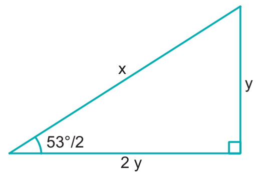
6.



Por Pitágoras:
$a^2 + (3a)^2 = (5\sqrt{10})^2$
$a^2 + 9a^2 = 250$
$10a^2 = 250$
$a^2 = 25 \Rightarrow a = 5$
$\Rightarrow x = 3a = 3(5) = 15$
$\Rightarrow y = a = 5$
$\therefore (x + y) = 15 + 5 = 20$

Clave E

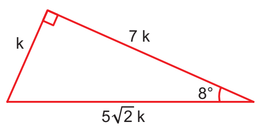
7.



Por Pitágoras:
$y^2 + (2y)^2 = x^2 \Rightarrow y^2 + 4y^2 = x^2$
$5y^2 = x^2 \Rightarrow \sqrt{5}\,y = x \Rightarrow \dfrac{x}{y} = \sqrt{5}$

Clave B

8. Del triángulo de 8° y 82°:



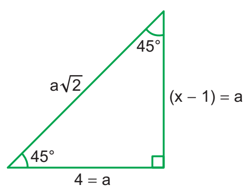
Entonces:
$a = 7k \;\wedge\; b = 5\sqrt{2}k$

$\Rightarrow \dfrac{b}{a} = \dfrac{5\sqrt{2}\,k}{7k} \Rightarrow \dfrac{b\sqrt{2}}{a} = \dfrac{5\sqrt{2}.\sqrt{2}}{7}$

$\therefore \dfrac{b\sqrt{2}}{a} = \dfrac{10}{7}$

Clave B

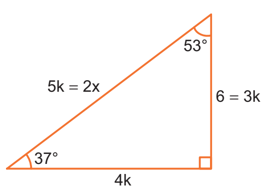
9.



Del gráfico: $a = 4$
$\Rightarrow (x - 1) = a \Rightarrow x - 1 = 4$
$\therefore x = 5$

Clave C

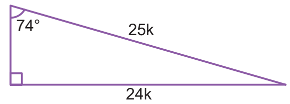
10.



Del gráfico: $3k = 6 \Rightarrow k = 2$
$\Rightarrow 2x = 5k$
$2x = 5(2)$
$2x = 10 \Rightarrow x = 5$

Clave D

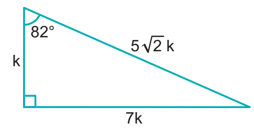
11. Del triángulo de 74° y 16°:



Entonces:
$a + 1 = 25k \wedge a = 24k$
Luego:
$24k + 1 = 25k \Rightarrow k = 1$
$\Rightarrow a = 24k$
$a = 24(1) \qquad \therefore 2a = 48$

Clave D

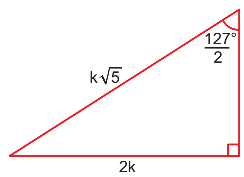
12. Del triángulo de 8° y 82°:



Entonces:
$5\sqrt{2}\,k = 35\sqrt{2} \Rightarrow k = 7$
Luego
$t = 7k = 7\,.\,7 \qquad \therefore t = 49$

Clave C

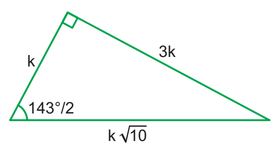
13. Del triángulo de 127°/2 y 53°/2:



Entonces:
$4\sqrt{5} = 2k \Rightarrow k = 2\sqrt{5}$
Luego
$m = k\sqrt{5} \Rightarrow m = 2\sqrt{5}.\sqrt{5} \qquad \therefore m = 10$

Clave A

14. Del triángulo $\dfrac{143°}{2}$ y $\dfrac{37°}{2}$:



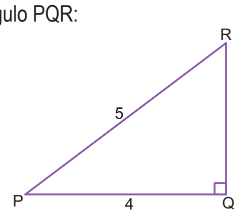
Entonces:
$k\sqrt{10} = 10\sqrt{5} \Rightarrow k\sqrt{2} = 10 \Rightarrow k = 5\sqrt{2}$
Luego:
$a + b = k + 3k$
$a + b = 4k$
$\dfrac{a+b}{2} = 2\,.\,5\sqrt{2}$
$\therefore \dfrac{a+b}{2} = 10\sqrt{2}$

Clave C

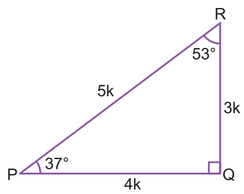
PRACTIQUEMOS
Nivel 1 (página 42) Unidad 3

Comunicación matemática

1. Del triángulo PQR:



Del triángulo de 37° y 53° tenemos:



Entonces:
m∠R = 53°
m∠P = 37°

En PQR:
PR = 5k = 5(1)
⇒ k = 1

Luego:
RQ = 3k = 3(1)
RQ = 3

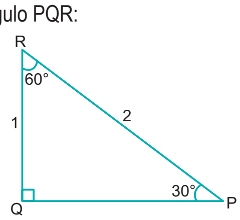
I. El triángulo de 37° y 53° es pitagórico. ...(Correcto)

II. La medida del lado RQ es igual a 3. ...(Correcto)

III. m∠R es igual a 53°. ...(Incorrecto)

Clave A

2. Del triángulo PQR:



Por el teorema de Pitágoras:
$PQ^2 + 1^2 = 2^2 \Rightarrow PQ^2 = 3 \Rightarrow PQ = \sqrt{3}$
Luego: PQR es un triángulo notable de 30° y 60°.

A) El triángulo notable de 30° y 60° no es pitagórico. ... (Incorrecto)

B) El triángulo de 30° y 60° es exacto. ... (Correcto)

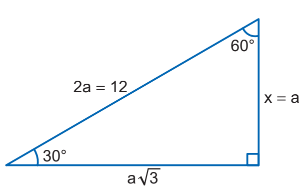
C) La medida de α es igual a 60°. ... (Correcto)

D) La medida de PQ es $\sqrt{3}$. ... (Correcto)

Clave A

Razonamiento y demostración

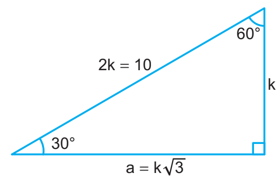
3.



Del gráfico:
$2a = 12 \Rightarrow a = 6$
$\Rightarrow x = a \qquad \therefore\ x = 6$

Clave C

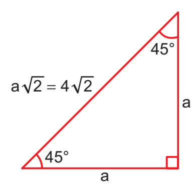
4.



Del gráfico:
$2k = 10 \Rightarrow k = 5$
$\Rightarrow a = k\sqrt{3} = (5)\sqrt{3} \qquad \therefore\ a = 5\sqrt{3}$

Clave B

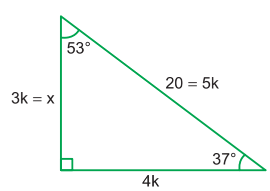
5.



Del gráfico:
$a\sqrt{2} = 4\sqrt{2} \qquad \therefore\ a = 4$

Clave E

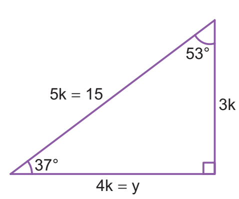
6.



Del gráfico:
$20 = 5k \Rightarrow 4 = k$
$\Rightarrow x = 3k = 3(4) = 12 \qquad \therefore\ x = 12$

Clave D

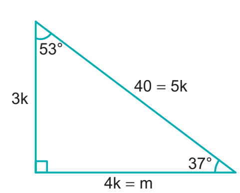
7.



Del gráfico:
$5k = 15 \Rightarrow k = 3$
$\Rightarrow y = 4k = 4(3) = 12 \qquad \therefore\ y = 12$

Clave A

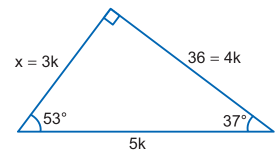
8.



Del gráfico:
$40 = 5k \Rightarrow 8 = k$
$\Rightarrow m = 4k = 4(8) = 32 \qquad \therefore\ m = 32$

Clave E

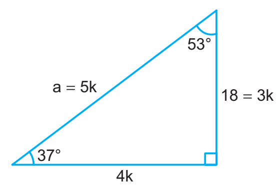
9.



Del gráfico:
$36 = 4k \Rightarrow 9 = k$
$\Rightarrow\ x = 3k = 3(9) = 27 \qquad \therefore\ x = 27$

Clave C

10.

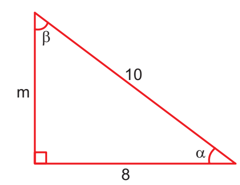


Del gráfico:
$18 = 3k \Rightarrow 6 = k$
$\Rightarrow a = 5k = 5(6) = 30 \qquad \therefore\ a = 30$

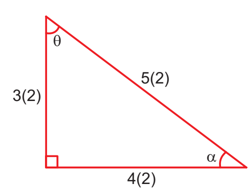
Clave B

Resolución de problemas

11. De los datos:



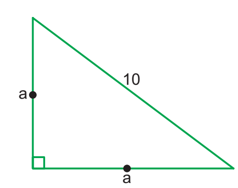
Del T. de Pitágoras:
$m^2 + 8^2 = 10^2 \Rightarrow m^2 = 100 - 64$
$m^2 = 36 \Rightarrow m = 6$
Luego, se observa:



Triángulo notable de 37° y 53°
∴ Menor ángulo: 37°

Clave D

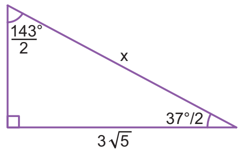
12. Del enunciado.
Triángulo notable de 45°:



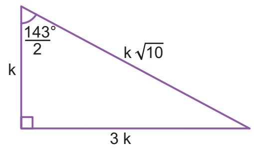
Luego:
$10 = a\sqrt{2} \qquad \therefore\ a = 5\sqrt{2}$

Clave B

13. Del enunciado:



Del triángulo de $\frac{37°}{2}$ y $\frac{143°}{2}$ tenemos:



Entonces:
$3k = 3\sqrt{5}$
$k = \sqrt{5}$

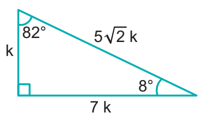
Luego:
$x = k\sqrt{10} \Rightarrow x = \sqrt{5}\,.\sqrt{10}$
$\therefore\ x = 5\sqrt{2}$

Clave B

Nivel 2 (página 42) Unidad 3

Comunicación matemática

14. El triángulo rectángulo de 8° y 82° no es pitagórico ya que los lados del triángulo no todos son enteros.



Clave C

15. I. El triángulo notable de 30° y 60° es exacto. ... (Falsa)

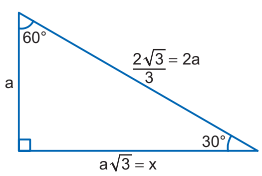
II. El triángulo notable de 16° y 74° es aproximado. ... (Falsa)

III. El triángulo rectángulo isósceles es el triángulo notable de 45° y no es pitagórico. ... (Falsa)

Clave D

Razonamiento y demostración

16.



Del gráfico:

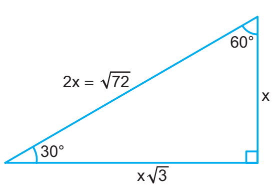
$2a = \frac{2\sqrt{3}}{3} \Rightarrow a = \frac{\sqrt{3}}{3}$

$\Rightarrow x = a\sqrt{3} = \left(\frac{\sqrt{3}}{3}\right)(\sqrt{3}) = 1$

$\therefore\ x = 1$

Clave D

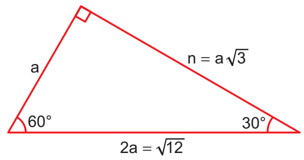
17.



Del gráfico:
$2x = \sqrt{72} = \sqrt{36\,.\,2} = 6\sqrt{2}$
$2x = 6\sqrt{2}$
$\therefore\ x = 3\sqrt{2}$

Clave A

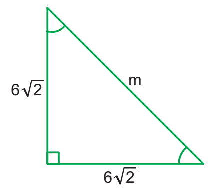
18.



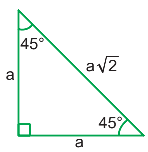
Del gráfico:
$2a = \sqrt{12} = 2\sqrt{3}$
$a = \sqrt{3}$
$\Rightarrow n = a\sqrt{3} = (\sqrt{3})(\sqrt{3})$
$\therefore n = 3$

Clave C

19.



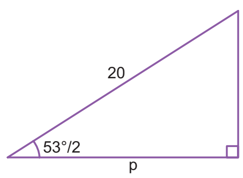
El ⊿ es notable de 45°:



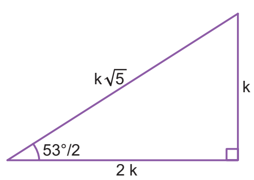
$\Rightarrow m = (6\sqrt{2})(\sqrt{2})$
$m = 6\,.\,2 = 12$
$\therefore\ m = 12$

Clave C

20.



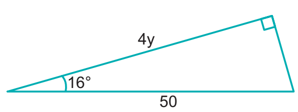
Del triángulo notable de $\frac{53°}{2}$ y $\frac{127°}{2}$:



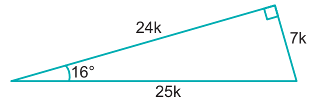
Luego:
$k\sqrt{5} = 20 \Rightarrow k = 4\sqrt{5}$
Entonces:
$p = 2k \Rightarrow p = 2(4\sqrt{5})$ $\qquad \therefore\ p = 8\sqrt{5}$

Clave E

21.



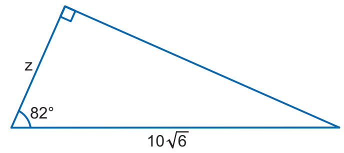
Del triángulo notable de 16° y 74°:



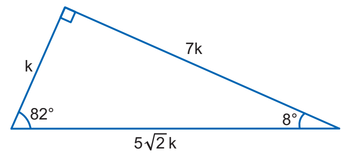
Luego:
$25k = 50 \Rightarrow k = 2$
Entonces:
$4y = 24k \Rightarrow 4y = 24(2)$ $\qquad \therefore\ y = 12$

Clave B

22.



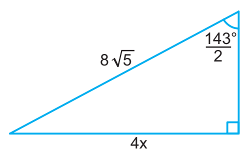
Del triángulo notable de 8° y 82°:



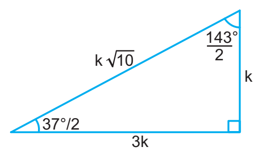
Luego:
$5\sqrt{2}\,k = 10\sqrt{6} \Rightarrow k = 2\sqrt{3}$
Entonces:
$z = k$ $\qquad \therefore\ z = 2\sqrt{3}$

Clave C

23.



Del triángulo notable de $\frac{37°}{2}$ y $\frac{143°}{2}$:

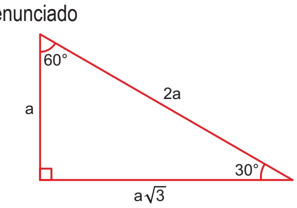


Luego:
$k\sqrt{10} = 8\sqrt{5} \Rightarrow k = 4\sqrt{2}$
Entonces:
$4x = 3k \Rightarrow 4x = 3(4\sqrt{2})$
$\therefore\ x = 3\sqrt{2}$

Clave D

Resolución de problemas

24. Del enunciado



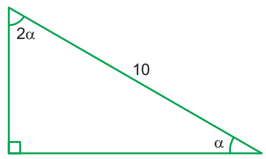
Mayor de los lados:
$2a = 6 \Rightarrow a = 3$

Mayor de los catetos:

$a\sqrt{3} = 3\sqrt{3} \qquad \therefore \ \dfrac{a\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{2}$

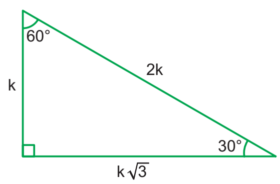
Clave B

25. Del enunciado, si α es el menor de los ángulos agudos:



$2\alpha + \alpha = 90° \Rightarrow 3\alpha = 90° \Rightarrow \alpha = 30°$

Dicho triángulo es notable de 30° y 60°:



Luego: $2k = 10 \Rightarrow k = 5$

Nos piden el menor de los catetos (k)

$\therefore \ k = 5$

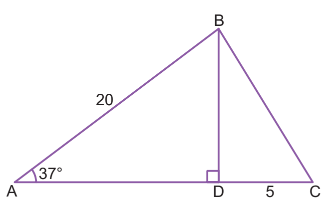
Clave B

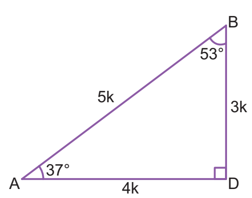
Nivel 3 (página 43) Unidad 3

Comunicación matemática

26. I. ABD es el triángulo notable de 37° y 53° el cual es aproximado, además los lados del triángulo son valores enteros, es decir, es pitagórico. ... (Verdadera)

II. Del triángulo:





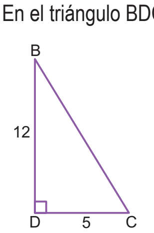
Entonces:
$5k = 20$
$k = 4$

Luego

$DB = 3k = 3 \ . \ 4 = 12$

$AD = 4k = 4 \ . \ 4 = 16$

En el triángulo BDC:



Por T. Pitágoras:

$BC^2 = 12^2 + 5^2$

$BC^2 = 169$

$BC = 13$

Los lados de BDC son enteros, entonces es pitagórico ... (Falsa)

III. De lo anterior:

$AD = 16$

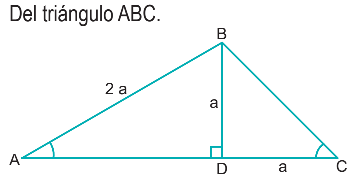
$AC = AD + DC$

$AC = 16 + 5$

$\therefore \ AC = 21$ ... (Verdadera)

Clave D

27. Del triángulo ABC.



I. ABD triángulo rectángulo de 30° y 60°:
Luego:

$m\angle BAD = 30° = 30° \ . \ \dfrac{\pi rad}{180°} = \dfrac{\pi}{6}$ rad

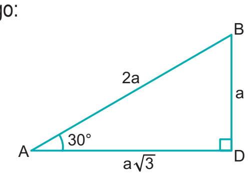
$\therefore \ m\angle BAD = \pi/6$ rad ... (Incorrecta)

II. BDC triángulo rectángulo de 45° luego:

$m\angle CBD = 45° = 45° \ . \ \dfrac{\pi rad}{180} = \dfrac{\pi}{4}$ rad

$\therefore \ m\angle CBD = \pi/4$ rad ... (Correcta)

III. El triángulo ABD notable de 30° y 60°:
Luego:

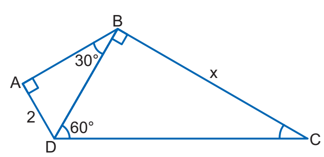


Los lados no son todos enteros, no es pitagórico. ... (Incorrecta)

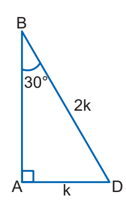
Clave D

Razonamiento y demostración

28.



En el triángulo notable DAB de 30° y 60°:

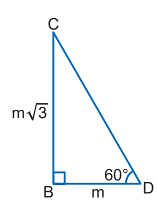


$AD = k = 2$

$\Rightarrow BD = 2k = 2 \ . \ (2)$

$BD = 4$

En el triángulo notable DBC de 30° y 60°:



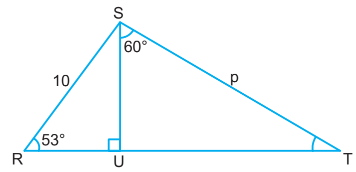
$BD = m = 4$

$\Rightarrow CB = m\sqrt{3} = 4\sqrt{3}$

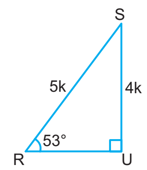
$\therefore \ x = CB = 4\sqrt{3}$

Clave B

29.



RUS ⊾ notable de 53° y 37°:



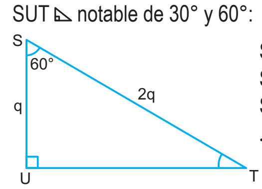
$5k = 10$

$k = 2$

$\Rightarrow SU = 4k = 4 \ . \ 2$

$SU = 8$

SUT ⊾ notable de 30° y 60°:



$SU = q = 8$

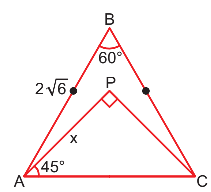
$ST = 2q = 2 \ . \ 8$

$ST = 16$

$\therefore \ p = ST = 16$

Clave E

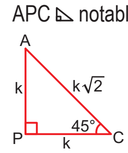
30.



ABC triángulo equilátero:

$AB = AC = 2\sqrt{6}$

APC ⊾ notable de 45°:



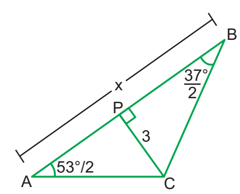
$AC = k\sqrt{2} = 2\sqrt{6}$

$\Rightarrow k = 2\sqrt{3}$

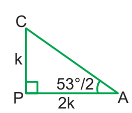
$\therefore \ x = k = 2\sqrt{3}$

Clave A

31.



APC ⊾ notable de $\dfrac{53°}{2}$ y $\dfrac{127°}{2}$:

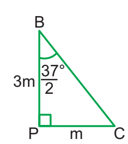


$PC = k = 3$

$AP = 2k = 2 \ . \ 3$

$AP = 6$

CPB ⊾ notable de $\dfrac{37°}{2}$ y $\dfrac{143°}{2}$:



$PC = m = 3$

$BP = 3m = 3 \ . \ 3$

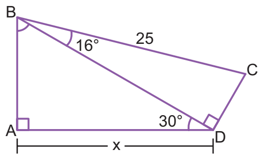
$BP = 9$

Luego: $x = AP + PB = 6 + 9$

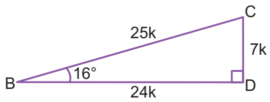
$\therefore\ x = 15$

Clave C

32.



BDC ◺notable de 16° y 74°:

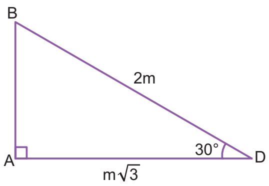


$BC = 25k = 25 \Rightarrow k = 1$

$BD = 24k = 24\ .\ 1$

$BD = 24$

BAD ◺notable de 30° y 60°:



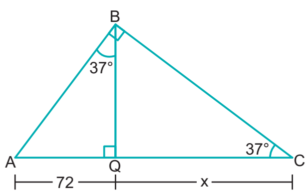
$BD = 2m = 24 \ \Rightarrow\ m = 12$

$AD = x = m\sqrt{3} = 12\sqrt{3}$

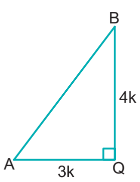
$\therefore\ x = 12\sqrt{3}$

Clave C

33.



AQB ◺notable 37° y 53°:



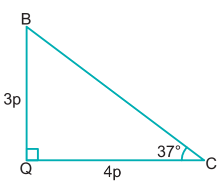
$AQ = 3k = 72$

$\Rightarrow k = 24$

$BQ = 4k = 4\ .\ 24$

$BQ = 96$

BQC ◺notable 37° y 53°:



$BQ = 3p = 96$

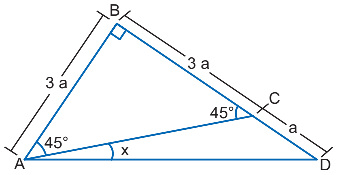
$\Rightarrow p = 32$

$x = 4p = 4\ .\ 32$

$\therefore\ x = 128$

Clave E

34.



Del gráfico.

ABC ◺notable 45°:

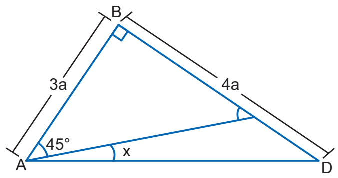
$AB = BC = 3a$

Luego:

$BD = BC + CD$

$BD = 3a + a$

$BD = 4a$



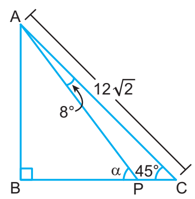
ABD ◺notable de 37° y 53°:

$m\angle BAD = 45° + x = 53°$

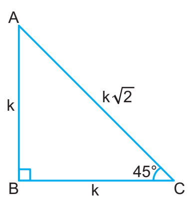
$\therefore\ x = 8°$

Clave B

35.



ABC ◺notable de 45°:



$AC = k\sqrt{2} = 12\sqrt{2}$

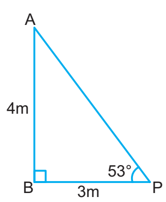
$\Rightarrow k = 12$

$AB = k = 12$

En ◺ABP:

$\alpha = 8° + 45° \Rightarrow\ \alpha = 53°$

ABP ◺notable de 53° y 37°:



$AB = 4\,m = 12$

$\Rightarrow m = 3$

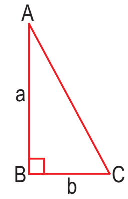
$BP = 3\,m = 3\ .\ 3$

$\therefore\ BP = 9$

Clave D

Resolución de problemas

36. Del enunciado sea el triángulo ABC rectángulo recto en B:

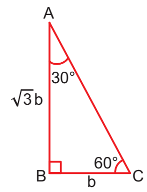


Por dato:

$\frac{a}{b} = \sqrt{3}$

$a = \sqrt{3}\,b$

Luego:



Se observa:

ABC ◺notable de 30° y 60°:

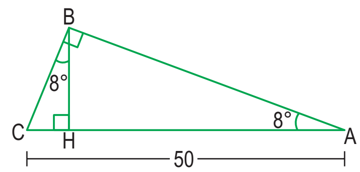
Finalmente; 60° es el mayor de los ángulos.

$\therefore\ 60°/2 = 30°$

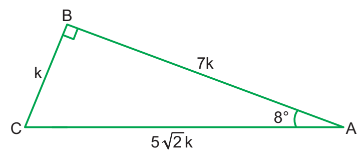
Clave C

37. Del enunciado; sea ABC un triángulo rectángulo.

BH: Altura relativa a la hipotenusa.



ABC ◺notable de 8° y 82°:

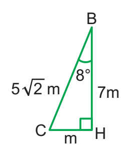


$AC = 5\sqrt{2}\,k = 50$

$\Rightarrow k = 5\sqrt{2}$

$BC = k = 5\sqrt{2}$

BHC ◺notable de 8° y 82°:



$BC = 5\sqrt{2}\,m = 5\sqrt{2}$

$\Rightarrow m = 1$

$BH = 7\,m = 7\ .\ 1$

$\therefore\ BH = 7$

Clave B

PRACTIQUEMOS

Nivel 1 (página 47) Unidad 3

Comunicación matemática

1. $\text{sen}30° = 1/2$ $\cos 37° = 4/5$

$\tan 8° = 1/7$

Clave D

2. A) $\text{sen}60° = \dfrac{\sqrt{3}}{2}$ ... (Incorrecta)

B) $\sec 45° = \sqrt{2}$ ... (Correcta)

C) $\cot 8° = \dfrac{1}{7}$ ... (Incorrecta)

D) $\text{sen}16° = \dfrac{24}{25}$ ... (Incorrecta)

Clave B

Razonamiento y demostración

3. $R = 6\sqrt{3}\,\text{sen}60°$

$R = 6\sqrt{3}\,.\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{6\sqrt{3}\,.\,\sqrt{3}}{2}$

$\therefore R = 9$

Clave B

4. $M = 10\text{sen}^2 45° - 2$

$M = 10\,.\left(\dfrac{\sqrt{2}}{2}\right)^2 - 2 = 10\,.\left(\dfrac{2}{4}\right) - 2$

$\therefore M = 3$

Clave A

5. $S = 8\text{sen}30° + 5\text{sen}37°$

$S = 8\,.\left(\dfrac{1}{2}\right) + 5\,.\left(\dfrac{3}{5}\right) = 4 + 3$

$\therefore S = 7$

Clave A

6. $S = 8\sqrt{3}\cos 30° + 2$

$S = 8\sqrt{3}\,.\left(\dfrac{\sqrt{3}}{2}\right) + 2$

$S = \dfrac{8\,.\,\sqrt{3}\,.\,\sqrt{3}}{2} + 2$

$\therefore S = 14$

Clave B

7. $N = 5\text{sen}37° + 10\text{sen}53°$

$N = 5\,.\left(\dfrac{3}{5}\right) + 10\,.\left(\dfrac{4}{5}\right)$

$N = 3 + 8$

$\therefore N = 11$

Clave D

8. $M = 3\sqrt{5}\cos\dfrac{53°}{2} + 4$

$M = 3\sqrt{5}\,.\dfrac{2}{\sqrt{5}} + 4$

$\therefore M = 10$

Clave B

9. $R = 7\tan 8° + 3\cot\dfrac{143°}{2} + 1$

$R = 7\,.\,1/7 + 3\,.\,1/3 + 1$

$\therefore R = 3$

Clave E

10. $T = 6\sqrt{3}\sec 30° \sec 16°$

$T = 6\,.\,\sqrt{3}\,.\,\dfrac{2}{\sqrt{3}}\,.\dfrac{25}{24}$

$\therefore T = 12{,}5$

Clave A

11. $M = \sqrt{3\tan\dfrac{37°}{2} + 3}$

$M = \sqrt{3\,.\dfrac{1}{3} + 3}$

$M = \sqrt{4}$

$\therefore M = 2$

Clave C

Resolución de problemas

12. Del enunciado se pide S, donde:

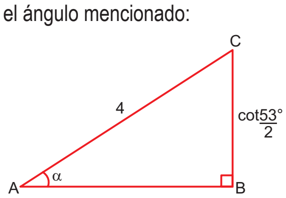
$S = \tan 82° - \tan 45°$

$S = 7 - 1$

$\therefore S = 6$

Clave A

13. Sea α el ángulo mencionado:

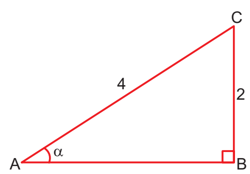


Datos:

$AC = 4$

$BC = \cot\dfrac{53°}{2} = 2$

Luego:



ABC triángulo rectángulo notable de 30° y 60°

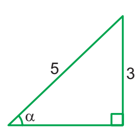
$\therefore \alpha = 30°$

Clave D

Nivel 2 (página 47) Unidad 3

Comunicación matemática

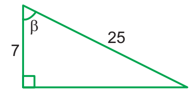
14. ▪ $\text{sen}\alpha = 3/5$



Notable de 37° y 53°.

$\therefore \alpha = 37°$

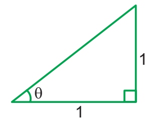
▪ $\cos\beta = 7/25$



Notable de 16° y 74°.

$\therefore \beta = 74°$

▪ $\tan\theta = 1$

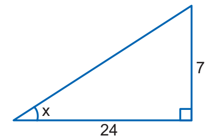


Notable de 45°.

$\therefore \theta = 45°$

Clave C

15. I. $\tan x = 7/24$, luego:

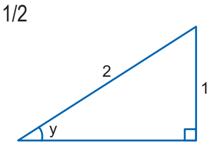


Triángulo notable de 16° y 74° (aproximado).

$\therefore x = 16°$

... (Falsa)

II. $\text{sen}y = 1/2$

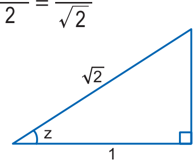


Triángulo notable de 30° y 60° (exacto).

$\therefore y = 30°$

... (Verdadera)

III. $\cos z = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$



Triángulo notable de 45°.

$\therefore z = 45°$

... (Verdadera)

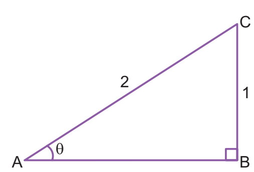
Clave A

Razonamiento y demostración

16. Dato: $\text{sen}\theta = \tan 53°/2 = 1/2$

$\text{sen}\theta = 1/2$

Luego:



ABC⊿ notable de 30° y 60°.

$\theta = 30°$

$\cot\theta = \cot 30°$

$\therefore \cot\theta = \sqrt{3}$

Clave E

17. $\text{sen}(x + \pi/6)\csc 3x = 1$

Por RT recíprocas.

$\Rightarrow\ x + \pi/6 = 3x$

$2x = \pi/6$

$x = \pi/12$

Luego:

$\tan 3x = \tan 3\pi/12 = \tan \pi/4$

$\therefore\ \tan 3x = 1$

Clave C

18. $M = \sqrt{3}\,\text{sen}60° + 4\sqrt{2}\,\text{sen}45° + \text{sen}30°$

$M = \sqrt{3}\left(\dfrac{\sqrt{3}}{2}\right) + 4\sqrt{2}\left(\dfrac{\sqrt{2}}{2}\right) + \left(\dfrac{1}{2}\right)$

$M = \dfrac{3}{2} + 4 + \dfrac{1}{2} = 6$

$\therefore M = 6$

Clave C

19. $M = \sqrt{8\sec 37° + 9\sec 53°}$

$M = \sqrt{8\left(\dfrac{5}{4}\right) + 9\left(\dfrac{5}{3}\right)}$

$M = \sqrt{10 + 15} = \sqrt{25}$

$\therefore M = 5$

Clave E

20. $E = \cot^2 30° + \sqrt{3}\cot 60° + 3\cot 45°$

$E = (\sqrt{3})^2 + \sqrt{3}\left(\dfrac{\sqrt{3}}{3}\right) + 3(1)$

$E = 3 + 1 + 3$

$\therefore E = 7$

Clave C

21. $A = \sqrt{2\sqrt{3}\cos 30° + \sqrt{3}\tan 30°}$

$A = \sqrt{2\sqrt{3}\left(\dfrac{\sqrt{3}}{2}\right) + \sqrt{3}\left(\dfrac{1}{\sqrt{3}}\right)}$

$A = \sqrt{3 + 1} = \sqrt{4}$

$\therefore\ A = 2$

Clave B

22. $Y = \sqrt{18\cot^2 60° - \sec 60°}$

$Y = \sqrt{18\left(\dfrac{1}{\sqrt{3}}\right)^2 - (2)}$

$Y = \sqrt{18\left(\dfrac{1}{3}\right) - 2} = \sqrt{6 - 2} = \sqrt{4}$

$\therefore Y = 2$

Clave C

23. $E = \sqrt{4\tan 37° + \sec^2 60° + 2}$

$E = \sqrt{4\left(\dfrac{3}{4}\right) + (2)^2 + 2}$

$E = \sqrt{3 + 4 + 2} = \sqrt{9}$

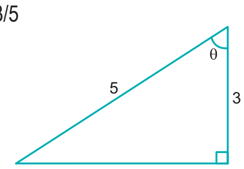
$\therefore E = 3$

Clave C

Resolución de problemas

24. Del enunciado:

$\cos\theta = 3/5$



Notable de 37° y 53°.

$\theta = 53°$

Luego:

$\alpha = 2(90° - 53°)$

$\alpha = 74°$

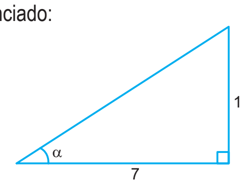
Finalmente:

$\csc\alpha = \csc 74°$

$\therefore\ \csc\alpha = 25/24$

Clave D

25. Del enunciado:



Notable de 8° y 82°.

$\alpha = 8°$

Luego:

$\text{sen}2\alpha = \text{sen}16°$

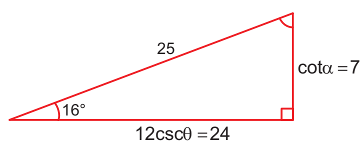
$\therefore\ \text{sen}2\alpha = 7/25$

Clave C

Nivel 3 (página 48) Unidad 3

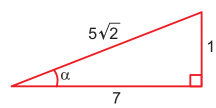
Comunicación matemática

26.



Del ⊿ notable de 16° y 74°.

- $\cot\alpha = 7$

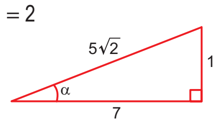


Es ⊿ notable de 8° y 82°.

$\Rightarrow\ \alpha = 8°$

- $12\csc\theta = 24$

$\csc\theta = 2$



Es ⊿ notable de 30° y 60°.

$\Rightarrow\ \theta = 30°$

I. $\cot\theta = \sqrt{3}$ ... (Verdadera)

II. $\alpha = 8°$ ... (Verdadera)

III. $90° - \theta = 90° - 30° = 60°$ ... (Falsa)

Clave B

27. $\cos(90° + b - 2a)\sec(a - 2b) = 1$

Razones trigonométricas recíprocas.

$90° + b - 2a = a - 2b$

$90° = 3a - 3b$

$a - b = 30°$

Luego:

I. $\sec(2a - 2b) = \sec 60° = 2$ ... (Verdadera)

II. $\tan(a - b + 15°) = \tan 45° = 1$... (Verdadera)

III. $\csc(a - b + 7°) = \csc 37° = 5/3$ ... (Falsa)

Clave E

Razonamiento y demostración

28. $y = 7\cot 82° + 4\sec^2 45° + 3\cot^2 30°$

$y = 7 \,.\, 1/7 + 4 \,.\, (\sqrt{2})^2 + 3(\sqrt{3})^2$

$y = 1 + 8 + 9$

$\therefore\ y = 18$

Clave B

29. $A = \sqrt{2 + 25\cos 74° + \tan 82°}$

$A = \sqrt{2 + 25 \,.\, \dfrac{7}{25} + 7}$

$A = \sqrt{16}$

$\therefore\ A = 4$

Clave A

30. $M = \sqrt{10}\,\text{sen}\dfrac{143°}{2} \,.\, \tan\dfrac{127°}{2} - 2\sqrt{5}\cos\dfrac{53°}{2}$

$M = \sqrt{10} \,.\, \dfrac{3}{\sqrt{10}} .2 - 2\sqrt{5}\,\dfrac{2}{\sqrt{5}}$

$M = 6 - 4$

$\therefore\ M = 2$

Clave E

31. $k = \sqrt{6}\tan 30°\text{sen}45° + 8\text{sen}82°\cos 45°\sec 37°$

$k = \sqrt{6}\,\dfrac{1}{\sqrt{3}} . \dfrac{1}{\sqrt{2}} + 8.\dfrac{7}{5\sqrt{2}} . \dfrac{1}{\sqrt{2}} . \dfrac{5}{4}$

$k = 1 + 7$

$\therefore\ k = 8$

Clave C

32. $\text{sen}20° = \cos(2\alpha - 4°)$

$\Rightarrow\ 20° + 2\alpha - 4° = 90°$

$2\alpha = 74°\ \Rightarrow\ \alpha = 37°$

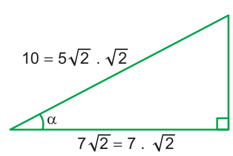
Luego:

$\tan \alpha/2 = \tan 37°/2 = 1/3$

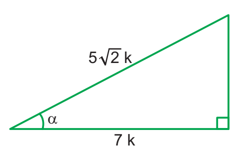
$\therefore\ \tan \alpha/2 = 1/3$

Clave E

33.



Luego:



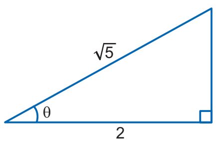
Triángulo rectángulo
notable de 8° y 82°
$\alpha = 8°$

Finalmente:
$\text{sen}(10\alpha - 6°) = \text{sen}(10 \,.\, 8 - 6°)$
$\text{sen}(10\alpha - 6°) = \text{sen}74°$
$\therefore\ \text{sen}(10\alpha - 6°) = 24/25$

Clave B

34. $\sqrt{5}\,\sec\theta = 5\text{sen}30°$

$$\sec\theta = \frac{\sqrt{5}}{2}$$



Del ⊿notable de $\frac{53°}{2}$ y $\frac{127°}{2}$:

$$\theta = \frac{53°}{2}$$

Luego:
$\text{sen}2\theta = \text{sen}\,2 \,.\, \frac{53°}{2}$
$\text{sen}2\theta = \text{sen}53°$
$\therefore\ \text{sen}2\theta = 4/5$

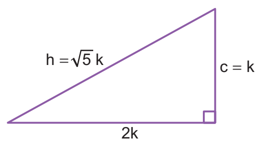
Clave D

35. $P = 5\text{sen}x\tan(6x - 3°)\sec(5x + 5°)$
Para $x = 8°$
$P = 5\text{sen}8°\tan(6 \,.\, 8° - 3°)\sec(5 \,.\, 8° + 5°)$
$P = 5\text{sen}8°\tan45°\sec45°$
$P = 5 \,.\, \frac{1}{5\sqrt{2}} . 1 . \sqrt{2}$
$\therefore\ P = 1$

Clave D

Resolución de problemas

36. Del enunciado:



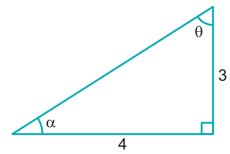
Triángulo rectángulo notable de $\frac{53°}{2}$ y $\frac{127°}{2}$.

$\frac{53°}{2}$: menor ángulo agudo.

Luego: $\text{sen}2(53°/2) = \text{sen}53°$
$\therefore\ \text{sen}53° = 4/5$

Clave A

37. Del enunciado: $\tan\theta = 4/3$



Triángulo notable de 37° y 53°.
$\theta = 53°$, $\alpha = 37°$
Luego:
$\cot \alpha/2 = \cot 37°/2$
$\therefore\ \cot \alpha/2 = 3$

Clave D

RESOLUCIÓN DE TRIÁNGULOS RECTÁNGULOS

APLICAMOS LO APRENDIDO (página 49) Unidad 3

1. ◺ABC: $BC = AC \,.\, \text{sen}\alpha$
$= m\text{sen}\alpha$
◺BDC: $x = BC \,.\, \text{sen}\alpha$
$x = (m\text{sen}\alpha)\text{sen}\alpha$
$x = m\text{sen}^2\alpha$

Clave C

2. ◺ABC: $CB = AC \,.\, \text{sen}\alpha$
$CB = m\text{sen}\alpha$
◺CBD: $x = CB\cot\beta$
$x = (m\text{sen}\alpha)\cot\beta$

Clave E

3. ◺AHB: $BH = AB\text{sen}\alpha$
$BH = a\text{sen}\alpha$
◺BHC: $x = BH\csc\beta$
$x = (a\text{sen}\alpha)\csc\beta$

Clave D

4. ◺ABC: $AC = AB\csc\phi$
$= a\csc\phi$
◺ACE: $x = AC\tan\theta$
$= a\csc\phi\tan\theta$

Clave A

5. ◺BAD: $BA = BD\text{sen}\theta$
$= m\text{sen}\theta$
También: $AD = BD\cos\theta$
$= m\cos\theta$
Luego: $A_{\square}\ ABCD = BA \,.\, AD$
$= m\text{sen}\theta m\cos\theta$
$= m^2\text{sen}\theta\cos\theta$

Clave A

6. $BC = AB\cot\theta$
$= m\cot\theta$
$AC = AB\csc\theta = m\csc\theta$
Luego $2p$◺ ABC $= AB + BC + AC$
$= m + m\cot\theta + m\csc\theta$
$= m(1 + \cot\theta + \csc\theta)$

Clave E

7. ◺ABC: $AB = AC\text{sen}\alpha$
$= m\text{sen}\alpha$
$m\angle ABH = \alpha$
$BH = AB\cos\alpha$
$BH = (m\text{sen}\alpha)\cos\alpha$

Clave C

8. ◺AHB: $BH = AH\tan\theta$
$= m\tan\theta$
$m\angle CBH = \theta$
$HC = BH\tan\theta$
$HC = (m\tan\theta)\tan\theta$
$= m\tan^2\theta$

Clave D

9. ◺ADB: $DB = AB\text{sen}\theta$
$= m\text{sen}\theta$
$m\angle DBE = \theta$
◺BED: $x = DB\text{sen}\theta$
$x = (m\text{sen}\theta)\text{sen}\theta = m\text{sen}^2\theta$

Clave E

10. $\triangle$BHA: $m\angle ABH = \theta$

$BH = AB\cos\theta$
$= a\cos\theta$

$\triangle$BHC: $x = BH\cot\theta$
$x = a\cos\theta\cot\theta$

Clave A

11. $\triangle$ABC: $CB = BA\tan\theta$
$= m\tan\theta$

$\triangle$BDC: $CD = CB\operatorname{sen}\alpha$
$= (m\tan\theta)\operatorname{sen}\alpha$

Clave E

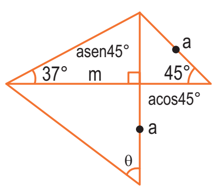
12. $\triangle$ABE: $BE = AE\cos\theta$
$= m\cos\theta$

$\triangle$BCE: $CE = BE\operatorname{sen}\theta$
$CE = m\cos\theta\operatorname{sen}\theta$

$\triangle$CDE: $CD = CE\operatorname{sen}\theta$
$= (m\cos\theta\operatorname{sen}\theta)\operatorname{sen}\theta$
$= m\cos\theta\operatorname{sen}^2\theta$

Clave B

13.



Del gráfico:

$\dfrac{m}{a\operatorname{sen}45°} = \cot 37°$

$m = a\operatorname{sen}45° \cdot \cot 37°$

$\tan\theta = \dfrac{m}{a}$

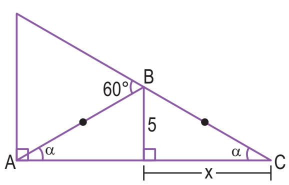
$\Rightarrow \tan\theta = \dfrac{a\operatorname{sen}45° \cdot \cot 37°}{a}$

$\tan\theta = \operatorname{sen}45° \cdot \cot 37° = \left(\dfrac{\sqrt{2}}{2}\right)\left(\dfrac{4}{3}\right)$

$\therefore \tan\theta = \dfrac{2\sqrt{2}}{3}$

Clave C

14.



El triángulo ABC es isósceles.

Del gráfico: $\alpha + \alpha = 60° \Rightarrow 2\alpha = 60° \Rightarrow \alpha = 30°$

$\Rightarrow \cot\alpha = \dfrac{x}{5} \Rightarrow x = 5\cot\alpha$

$x = 5\cot 30° = 5(\sqrt{3}) \qquad \therefore x = 5\sqrt{3}$

Clave C

PRACTIQUEMOS

Nivel 1 (página 51) Unidad 3

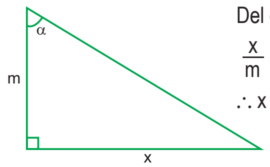
Comunicación matemática

1.

2.

Razonamiento y demostración

3.



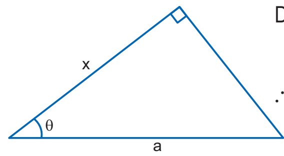
Del gráfico:

$\dfrac{x}{m} = \tan\alpha$

$\therefore x = m\tan\alpha$

Clave C

4.



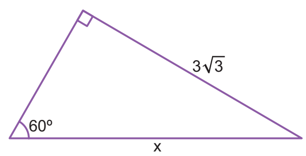
Del gráfico:

$\dfrac{x}{a} = \cos\theta$

$\therefore x = a\cos\theta$

Clave E

5.



Del gráfico:

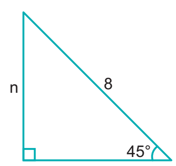
$\dfrac{x}{3\sqrt{3}} = \csc 60°$

$x = 3\sqrt{3}\ \csc 60°$

$x = 3\sqrt{3}\left(\dfrac{2}{\sqrt{3}}\right) = 6 \qquad \therefore x = 6$

Clave A

6.



Del gráfico:

$\dfrac{n}{8} = \operatorname{sen}45°$

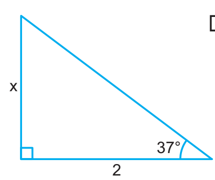
$n = 8\operatorname{sen}45°$

$n = 8\left(\dfrac{\sqrt{2}}{2}\right) = 4\sqrt{2}$

$\therefore n = 4\sqrt{2}$

Clave D

7.



Del gráfico:

$\dfrac{x}{2} = \tan 37°$

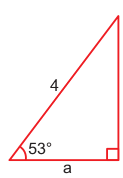
$x = 2\tan 37°$

$x = 2\left(\dfrac{3}{4}\right) = \dfrac{3}{2} = 1{,}5$

$\therefore x = 1{,}5$

Clave E

8.



Del gráfico:

$\dfrac{a}{4} = \cos 53° \Rightarrow a = 4\cos 53°$

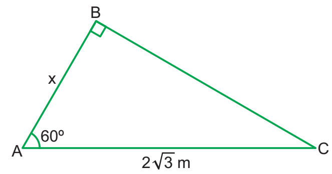
$a = 4\left(\dfrac{3}{5}\right) = \dfrac{12}{5}$

$\therefore a = \dfrac{12}{5}$

Clave C

Resolución de problemas

9.

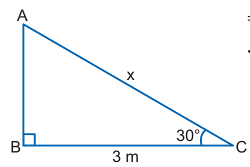


$\Rightarrow x = 2\sqrt{3}\ .\cos 60°$

$\therefore x = \sqrt{3}$ m

Clave E

10.



$\Rightarrow x = 3\sec 30°$

$\therefore x = 2\sqrt{3}$ m

Clave C

Nivel 2 (página 52) Unidad 3

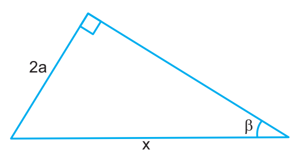
Comunicación matemática

11.

12.

Razonamiento y demostración

13.

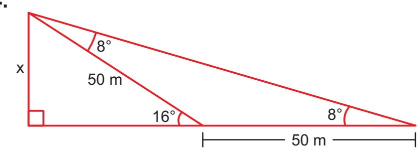


Del gráfico:

$\dfrac{x}{2a} = \csc\beta \qquad \therefore x = 2a\csc\beta$

Clave A

14.

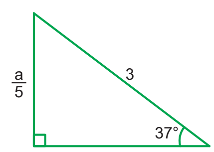


$\Rightarrow x = 50\operatorname{sen}16°$

$\therefore x = 14$ m

Clave A.

15.



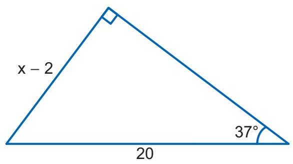
En el gráfico:

$\dfrac{\left(\frac{a}{5}\right)}{3} = \operatorname{sen}37° \Rightarrow \dfrac{a}{5} = 3\operatorname{sen}37°$

$a = 15\text{sen}37° = 15\left(\frac{3}{5}\right)$ $\therefore\ a = 9$

Clave A

16.



Del gráfico:

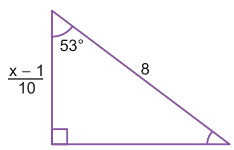
$\frac{(x-2)}{20} = \text{sen}37° \Rightarrow x - 2 = 20\text{sen}37°$

$x - 2 = 20\left(\frac{3}{5}\right) = 12 \Rightarrow x - 2 = 12$

$\therefore\ x = 14$

Clave A

17.



Del gráfico:

$\frac{\left(\frac{x-1}{10}\right)}{8} = \cos 53°$

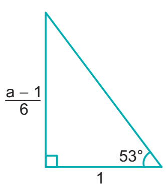
$\frac{x-1}{10} = 8\cos 53° = 8\left(\frac{3}{5}\right)$

$\frac{x-1}{10} = \frac{24}{5} \Rightarrow x - 1 = 48$

$\therefore x = 49$

Clave B

18.



$\frac{\left(\frac{a-1}{6}\right)}{1} = \tan 53°$

$a - 1 = 6\tan 53°$

$a = 6\tan 53° + 1$

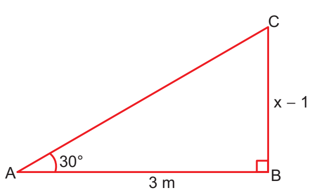
$a = 6\left(\frac{4}{3}\right) + 1$

$\therefore\ a = 9$

Clave D

Resolución de problemas

19.

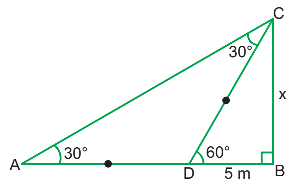


$\Rightarrow\ x - 1 = 3\tan 30°$

$\therefore\ x = \sqrt{3} + 1$

Clave D

20.



$\Rightarrow\ x = 5\tan 60°$ $\therefore\ x = 5\sqrt{3}$

Clave B

Nivel 3 (página 52) Unidad 3

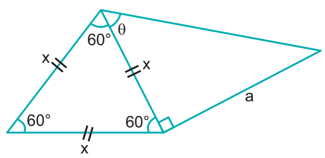
Comunicación matemática

21.

22.

Razonamiento y demostración

23.

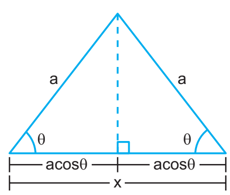


Del gráfico:

$\frac{x}{a} = \cot\theta$ $\therefore\ x = a\cot\theta$

Clave C

24. Trazamos la altura del triángulo isósceles.

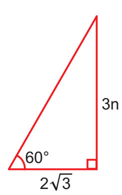


Del gráfico:

$x = a\cos\theta + a\cos\theta$ $\therefore\ x = 2a\cos\theta$

Clave A

25.



Del gráfico:

$\frac{3n}{2\sqrt{3}} = \tan 60°$

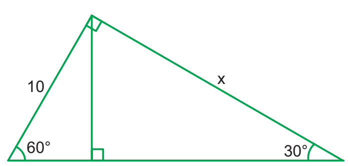
$3n = 2\sqrt{3}\tan 60°$

$3n = 2\sqrt{3}\,(\sqrt{3}) \Rightarrow 3n = 6$

$\therefore n = 2$

Clave C

26.



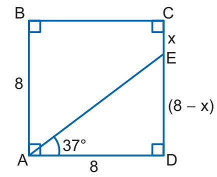
Del gráfico:

$\frac{x}{10} = \tan 60° \Rightarrow x = 10\tan 60°$

$x = 10(\sqrt{3})$ $\therefore x = 10\sqrt{3}$

Clave E

27.



Piden: CE = x

Del gráfico:

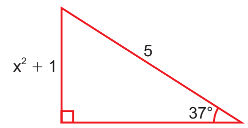
$\frac{8-x}{8} = \tan 37° \Rightarrow 8 - x = 8\tan 37°$

$x = 8 - 8\tan 37° \Rightarrow x = 8 - 8\left(\frac{3}{4}\right)$

$x = 8 - 6$ $\therefore\ x = 2$

Clave E

28.



Del gráfico:

$\frac{(x^2+1)}{5} = \text{sen}37° \Rightarrow x^2 + 1 = 5\text{sen}37°$

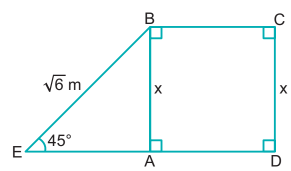
$x^2 + 1 = 5\left(\frac{3}{5}\right) \Rightarrow x^2 + 1 = 3$

$x^2 = 2$ $\therefore\ x = \sqrt{2}$

Clave B

Resolución de problemas

29.

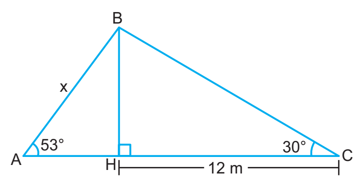


$\Rightarrow\ x = \sqrt{6}\,\text{sen}45° = \sqrt{6}\,.\,\frac{1}{\sqrt{2}}$

$\therefore\ x = \sqrt{3}$ m

Clave D

30.



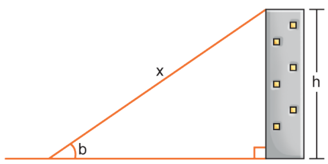
- $BH = 12\tan 30°$
  $BH = 4\sqrt{3}$
- $x = BH\csc 53° = 4\sqrt{3}\,.\,\frac{5}{4}$

$\therefore\ x = 5\sqrt{3}$ m

Clave D

APLICAMOS LO APRENDIDO (página 54) Unidad 3

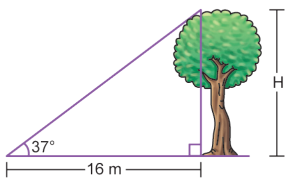
1.



Del gráfico: $\frac{x}{h} = \csc b$ $\therefore x = h\csc b$

Clave E

2.



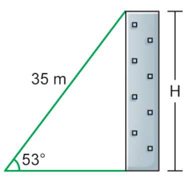
Del gráfico:

$\tan 37° = \frac{H}{16} \Rightarrow \frac{3}{4} = \frac{H}{16}$

$H = \frac{16.3}{4} = 12$ $\therefore H = 12$ m

Clave C

3.



Del gráfico:

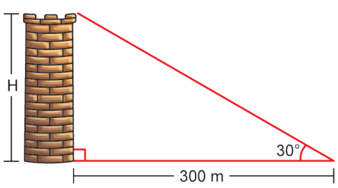
$\underbrace{\text{sen}53°}_{\frac{4}{5}} = \frac{H}{35}$

$\Rightarrow \frac{4}{5} = \frac{H}{35} \Rightarrow H = \frac{35.4}{5} = 28$

$\therefore H = 28$ m

Clave B

4.



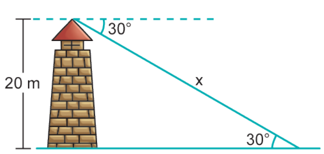
Del gráfico: $\underbrace{\tan 30°}_{\frac{\sqrt{3}}{3}} = \frac{H}{300}$

$\Rightarrow \frac{\sqrt{3}}{3} = \frac{H}{300} \Rightarrow H = \frac{300\sqrt{3}}{3} = 100\sqrt{3}$

$\therefore H = 100\sqrt{3}$ m

Clave C

5.

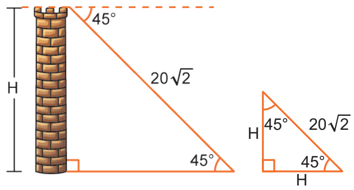


Del gráfico:

$x = 20\csc 30° \Rightarrow x = 20(2)$ $\therefore x = 40$ m

Clave C

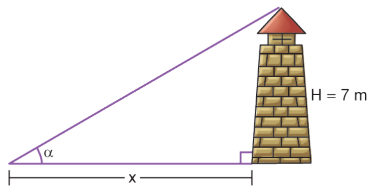
6.



$H\sqrt{2} = 20\sqrt{2}$ $\therefore H = 20$

Clave C

7.



$\Rightarrow \tan\alpha = \frac{H}{x}$

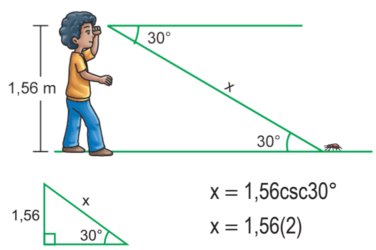
Por dato: $\tan\alpha = \frac{1}{4} \Rightarrow \frac{H}{x} = \frac{1}{4}$

Reemplazando el valor de H:

$\frac{7}{x} = \frac{1}{4} \Rightarrow 7.4 = x$ $\therefore x = 28$ m

Clave B

8.



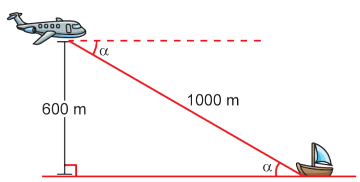
$x = 1{,}56\csc 30°$

$x = 1{,}56(2)$

$x = 3{,}12$ m

Clave B

9.



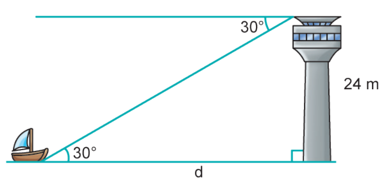
Del gráfico:

$\text{sen}\alpha = \frac{600}{1000} \Rightarrow \text{sen}\alpha = \frac{3}{5}$

$\text{sen}37° = \frac{3}{5}$ $\therefore \alpha = 37°$

Clave B

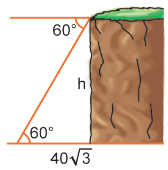
10.



$d = 24\cot 30° \Rightarrow d = 24\sqrt{3}$

Clave A

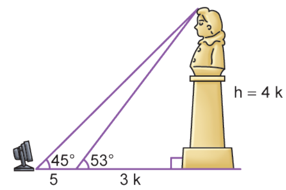
11.



$h = 40\sqrt{3}\tan 60°$
$= 40\sqrt{3}(\sqrt{3}) = 120$ m

Clave E

12.

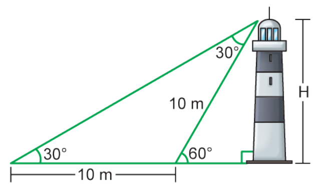


Del gráfico: $5 + 3k = 4k \Rightarrow k = 5$

$h = 4(5) = 20$ m

Clave C

13.



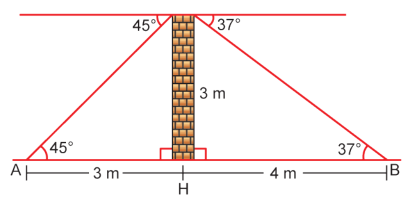
Del gráfico: $\underbrace{\text{sen}60°}_{} = \frac{H}{10}$

$\frac{\sqrt{3}}{2} = \frac{H}{10} \Rightarrow H = \frac{10\sqrt{3}}{2}$

$\therefore H = 5\sqrt{3}$ m

Clave D

14.



Usando los triángulos rectángulos de 45°; 37° y 53°:

AH = 3 m ∧ HB = 4 m

$\therefore$ AB = 3 + 4 = 7 m

Clave B

PRACTIQUEMOS
Nivel 1 (página 56) Unidad 3

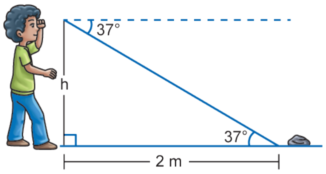
Comunicación matemática

1. Pierre de Fermat (1601–1665): Matemático francés, recordado por sus aportes a la teoría de número y la publicación del teorema de Fermat.

2.

Razonamiento y demostración

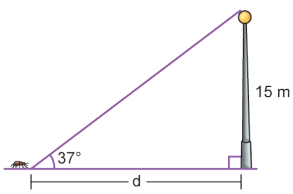
3.



h = 2tan37° ∴ h = 1,5 m

Clave A.

4.

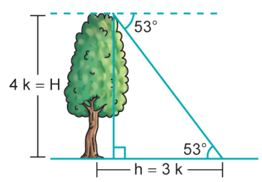


d = 15cot37° ∴ d = 20 m

Clave D

Resolución de problemas

5.



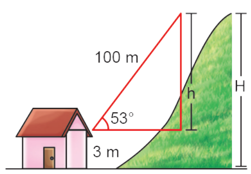
Del dato: H – h = 1

(4k) – (3k) = 1 ⇒ k = 1

Piden: H = 4k = 4(1) ∴ H = 4 m

Clave D

6.



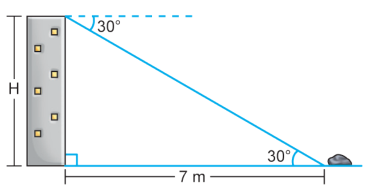
$\Rightarrow$ h = 100sen53°

$h = 100\dfrac{(4)}{5} = 80$

∴ H = h + 3 = 83 m

Clave A

7.

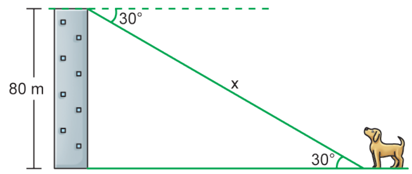


Del gráfico: $\dfrac{H}{7} = \tan 30° \Rightarrow H = 7\tan 30°$

$H = 7\left(\dfrac{\sqrt{3}}{3}\right)$ $\therefore H = \dfrac{7\sqrt{3}}{3}$ m

Clave C

8.



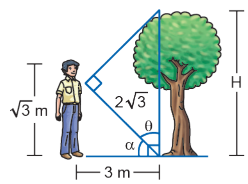
Del gráfico:

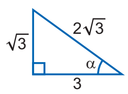
$\dfrac{x}{80} = \csc 30° \Rightarrow x = 80\csc 30°$

x = 80(2) ∴ x = 160 m

Clave B

9.





α notable

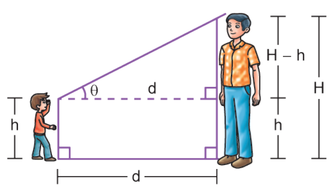
α = 30° ⇒ θ = 60°

$H = 2\sqrt{3}\sec\theta \Rightarrow H = 2\sqrt{3}\sec 60° = 4\sqrt{3}$

$\therefore H = 4\sqrt{3}$ m

Clave C

10.



Del gráfico:

$\dfrac{d}{H-h} = \cot\theta$ ∴ d = (H – h)cotθ

Clave D

Nivel 2 (página 57) Unidad 3

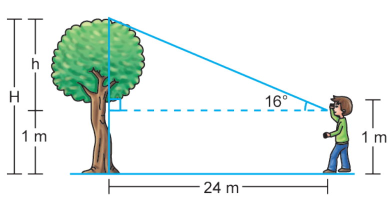
Comunicación matemática

11.

12.

Razonamiento y demostración

13.

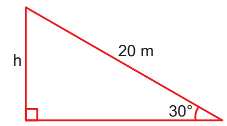


h = 24tan16° ⇒ h = 7 m

∴ H = h + 1 = 8 m

Clave D

14.



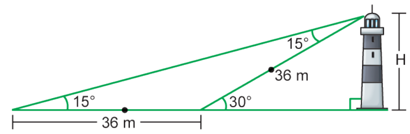
h = 20sen30°

∴ h = 10 m

Clave A

Resolución de problemas

15.



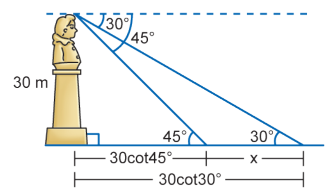
Del gráfico:

$\dfrac{H}{36} = \text{sen}30° \Rightarrow H = 36\text{sen}30° = 36\left(\dfrac{1}{2}\right)$

∴ H = 18 m

Clave E

16.



Del gráfico:

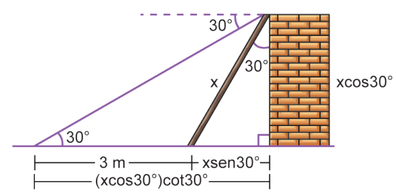
30cot30° = 30cot45° + x

$30(\sqrt{3}) = 30(1) + x \Rightarrow 30(1{,}73) = 30 + x$

51,9 = 30 + x ∴ x = 21,9 m

Clave A

17.



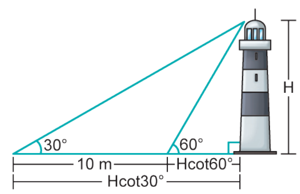
Del gráfico: (xcos30°)cot30° = xsen30° + 3

$x\,.\left(\dfrac{\sqrt{3}}{2}\right)(\sqrt{3}) = x\left(\dfrac{1}{2}\right) + 3$

$\dfrac{3}{2}x = \dfrac{x}{2} + 3$ ∴ x = 3 m

Clave A

18.



Del gráfico:

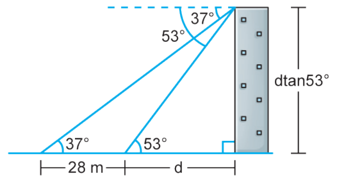
Hcot60° + 10 = Hcot30° ⇒ H(cot30° – cot60°) = 10

$H\left(\sqrt{3} - \dfrac{\sqrt{3}}{3}\right) = 10 \Rightarrow H\left(\dfrac{2\sqrt{3}}{3}\right) = 10$

$H = 5\sqrt{3}$

Clave C

19.



Del gráfico:

$\dfrac{d\tan 53°}{28+d} = \tan 37°$

$d\tan 53° = \tan 37°(28 + d)$

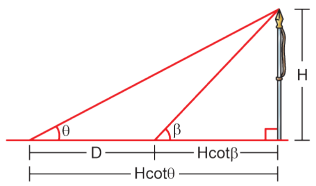
$d\left(\dfrac{4}{3}\right) = \left(\dfrac{3}{4}\right)(28+d)$

$\dfrac{7d}{12} = 21 \Rightarrow d = 36$ m

$\Rightarrow$ Velocidad $= \dfrac{d}{t} = \dfrac{36\text{ m}}{6\text{ s}} = 6\ \dfrac{\text{m}}{\text{s}}$

Clave B

20.



Del gráfico:

$D + H\cot\beta = H\cot\theta$

$D = H\cot\theta - H\cot\beta \Rightarrow D = H(\cot\theta - \cot\beta)$

$\therefore H = \dfrac{D}{\cot\theta - \cot\beta}$

Clave B

Nivel 3 (página 57) Unidad 3

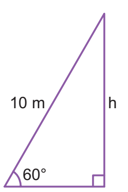
Comunicación matemática

21.

22. Del gráfico: I) F; II) V; III) V

Razonamiento y demostración

23.

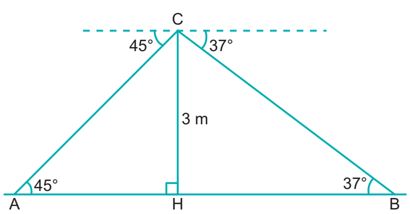


$h = 10\text{sen}60°$

$\therefore\ h = 5\sqrt{3}$

Clave C

24.



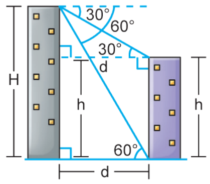
- $AH = 3\cot 45°$
  $AH = 3$ m
- $HB = 3\cot 37°$
  $HB = 4$ m

$\therefore AB = 4 + 3 = 7$ m

Clave E

Resolución de problemas

25.



Del gráfico:

$H = d\tan 60°$

$H - h = d\tan 30°$

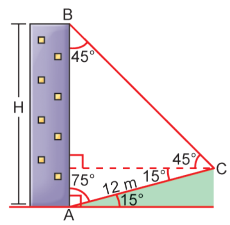
$\Rightarrow \dfrac{H-h}{H} = \dfrac{d\tan 30°}{d\tan 60°} = \dfrac{\left(\frac{\sqrt{3}}{3}\right)}{\sqrt{3}} = \dfrac{1}{3}$

$\Rightarrow \dfrac{H-h}{H} = \dfrac{1}{3} \Rightarrow 3H - 3h = H \Rightarrow 2H = 3h$

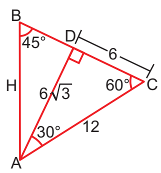
$\therefore \dfrac{H}{h} = \dfrac{3}{2}$

Clave C

26.



En el $\triangle ABC$:



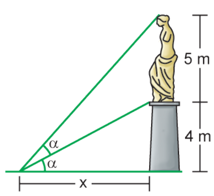
En el triángulo notable de 30° y 60°:
$AD = 6\sqrt{3}$

En el triángulo notable de 45°:

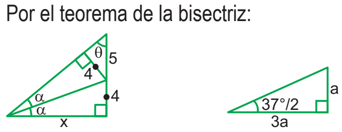
$H = AD\,.\,\sqrt{2} \Rightarrow H = 6\sqrt{3}\,.\,\sqrt{2}\ \therefore\ H = 6\sqrt{6}$ m

Clave C

27.



Por el teorema de la bisectriz:



$\Rightarrow$ El ángulo θ es notable: $\theta = 53°$

Además: $2\alpha + \theta = 90° \Rightarrow \alpha = \dfrac{37°}{2}$

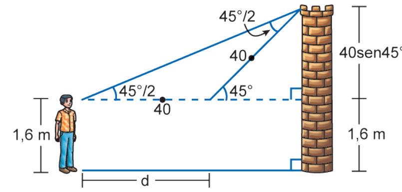
Luego: $a = 4$

$\Rightarrow x = 3a = 3(4) = 12$

$\therefore x = 12$ m

Clave B

28.

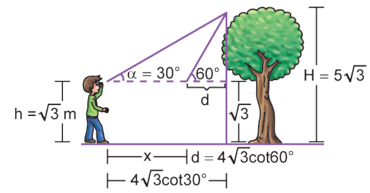


$\Rightarrow$ La altura de la torre $= \underbrace{40\text{sen}45°}_{(28,2)} + 1{,}6$

$= 29{,}8$ m

Clave C

29.



Del dato:

$\alpha = 90° - 60° = 30°$

$H = 5\sqrt{3}$ m

$h = 1{,}73\text{ m} \approx \sqrt{3}$ m

Del gráfico:

$x + d = x + 4\sqrt{3}\cot 60° = 4\sqrt{3}\cot 30°$

$x = 4\sqrt{3}(\cot 30° - \cot 60°)$

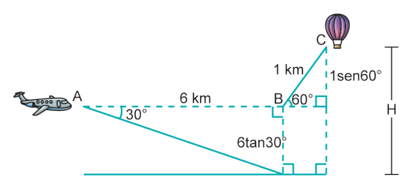
$x = 4\sqrt{3}\left(\sqrt{3} - \dfrac{\sqrt{3}}{3}\right) = \dfrac{4\sqrt{3}}{3}(3\sqrt{3} - \sqrt{3})$

$x = \dfrac{4\sqrt{3}}{3}\,.\,(2\sqrt{3}) = 8$

$\therefore x = 8$ m (aprox.)

Clave A

30.



Del dato, para el tramo AB:

$V = 180$ km/h $= 3$ km/min

$t_{vuelo} = 2$ min

$\Rightarrow AB = (3\text{ km/min})(2\text{ min}) = 6$ km

Para el tramo BC:

$BC = 1000\text{ m} = 1$ km

Del gráfico:

$H = 6\tan 30° + 1\text{sen}60°$

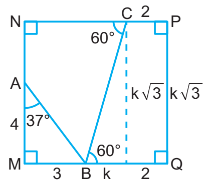
$H = 2\sqrt{3} + 0{,}5\sqrt{3} = 2{,}5\sqrt{3}$

$\therefore H = 2{,}5\sqrt{3}$ km

Clave B

MARATÓN MATEMÁTICA (página 59)

1.



$k\sqrt{3} = k + 5$

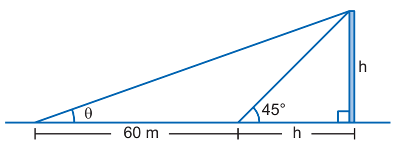
$k(\sqrt{3} - 1) = 5$

$\therefore k = \dfrac{5}{\sqrt{3} - 1}$

$k = \dfrac{5}{2}(\sqrt{3} - 1)$

Clave C

2.



Si $\operatorname{sen}\theta = \dfrac{1}{\sqrt{37}} \Rightarrow \tan\theta = \dfrac{1}{6}$

Luego:

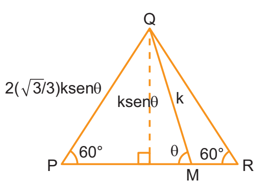
$\tan\theta = \dfrac{1}{6} = \dfrac{h}{60 + h} \Rightarrow 6h = 60 + h$

$5h = 60$

$\therefore\ h = 12 \text{ m}$

Clave E

3. $k = \dfrac{\sqrt{3}}{2}$



$$A_{\Delta PQR} = \frac{(k\operatorname{sen}\theta) \times 2\left(\frac{\sqrt{3}}{3}\right)k\operatorname{sen}\theta}{2} k\operatorname{sen}\theta$$

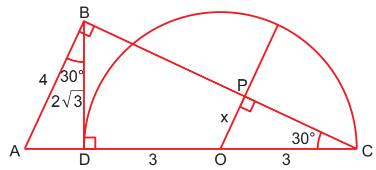
$$A_{\Delta PQR} = \left(\frac{\sqrt{3}}{3}\right)k^2\operatorname{sen}^2\theta$$

$$A_{\Delta PQR} = \left(\frac{\sqrt{3}}{3}\right)\left(\frac{3}{4}\right)\operatorname{sen}^2\theta$$

$$\therefore A_{\Delta PQR} = \frac{\sqrt{3}}{4}\operatorname{sen}^2\theta$$

Clave B

4. Del gráfico tenemos:

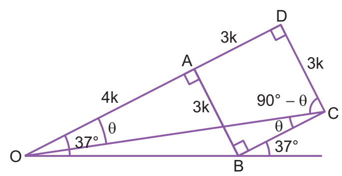


$\operatorname{sen}30° = \dfrac{x}{3} \Rightarrow \dfrac{1}{2} = \dfrac{x}{3}$

$\therefore\ x = \dfrac{3}{2}$

Clave D

5.



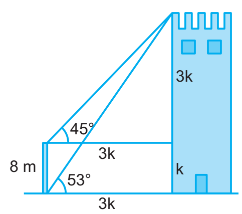
Calculamos M:

$M = \cot\theta - 2 = \dfrac{7k}{3k} - 2$

$\therefore\ M = \dfrac{1}{3}$

Clave E

6.



Del gráfico tenemos:

$k = 8 \text{ m}$

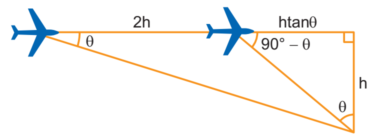
Nos piden:

$H_{torre} = 4k \Rightarrow H_{torre} = 4\ (8 \text{ m})$

$\therefore\ H_{torre} = 32 \text{ m}$

Clave A

7.



Del gráfico tenemos:

$\tan\theta = \dfrac{h}{2h + h\tan\theta}$

$\tan\theta\ (2 + \tan\theta) = 1$

$2 + \tan\theta = \dfrac{1}{\tan\theta}$

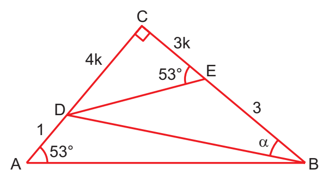
$2 = \cot\theta - \tan\theta$

$2 = \underbrace{\cot\theta - \tan\theta}_{R}$

$\therefore\ R = 2$

Clave C

8.



Del gráfico tenemos:

$\tan 53° = \dfrac{4}{3} = \dfrac{3 + 3k}{4k + 1} \Rightarrow 16k + 4 = 9 + 9k$

$7k = 5$

$k = \dfrac{5}{7}$

Nos piden:

$$\tan\alpha = \frac{4k}{3k + 3} = \frac{\frac{20}{7}}{\frac{15}{7} + 3} = \frac{\frac{20}{7}}{\frac{36}{7}}$$

$\therefore\ \tan\alpha = \dfrac{5}{9}$

Clave B

9. En (1) los ángulos son recíprocos:

$\Rightarrow\ 2x = y - x$

$3x = y$

En (2) los ángulos son complementarios:

$\Rightarrow\ 2x + y = 90°$

$2x + 3x = 90° \Rightarrow 5x = 90°$

$x = 18°$

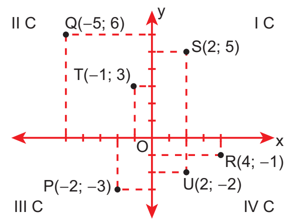
$\Rightarrow\ y = 54°$

$\therefore\ x + y = 72°$

Clave C

APLICAMOS LO APRENDIDO (página 61) Unidad 4

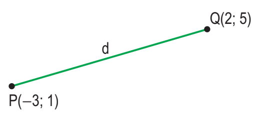
1. Ubicamos los puntos en el plano cartesiano:



$\therefore$ R y U $\in$ IVC

Clave D

2.



$d = \sqrt{(2-(-3))^2+(5-1)^2}$

$d = \sqrt{5^2+4^2} = \sqrt{41}$ $\qquad \therefore d = \sqrt{41}$

Clave D

3. $d = \sqrt{(x-4)^2+(-2-2x)^2}$

$5 = \sqrt{(x-4)^2+(2+2x)^2}$

$25 = x^2 - 8x + 16 + 4 + 8x + 4x^2$

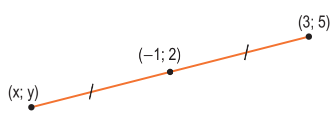
$25 = 5x^2 + 20$

$5 = 5x^2$

$1 = x^2$ $\qquad \therefore x = \pm 1$

Clave D

4.



Por propiedad:

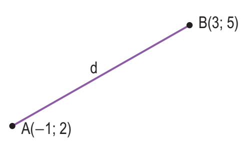
$-1 = \dfrac{x+3}{2} \Rightarrow x = -5$

$2 = \dfrac{y+5}{2} \Rightarrow y = -1$

$\Rightarrow (x + y) = -5 + (-1)$ $\qquad \therefore (x + y) = -6$

Clave B

5.



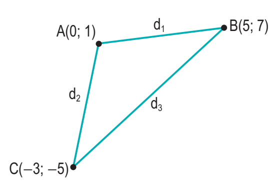
$d = \sqrt{(3-(-1))^2+(5-2)^2}$

$d = \sqrt{4^2+3^2} = \sqrt{25}$

$\therefore d = 5$

Clave B

6.



$d_1 = \sqrt{(5-0)^2+(7-1)^2}$

$d_1 = \sqrt{5^2+6^2} = \sqrt{61} = 7{,}8$

$d_2 = \sqrt{(0-(-3))^2+(1-(-5))^2}$

$d_2 = \sqrt{3^2+6^2} = \sqrt{45} = 6{,}7$

$d_3 = \sqrt{(5-(-3))^2+(7-(-5))^2}$

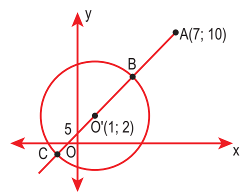
$d_3 = \sqrt{8^2+12^2} = \sqrt{208} = 14{,}4$

$\Rightarrow d_2 < d_1 < d_3$

$\therefore$ La menor distancia es $d_2 = \sqrt{45} = 3\sqrt{5}$

Clave B

7.



Hallamos la distancia AO':

$AO' = \sqrt{(7-1)^2+(10-2)^2}$

$= \sqrt{6^2+8^2} = \sqrt{100} = 10$

$\Rightarrow AO' = 10$

Del gráfico:

AB: mínima distancia a la circunferencia.

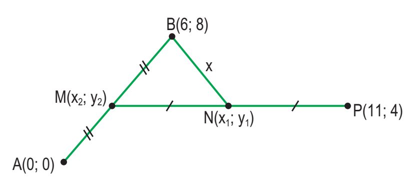
AC: máxima distancia a la circunferencia.

$AB = \underbrace{AO'}_{10} - \underbrace{BO'}_{5} \quad \wedge \quad AC = \underbrace{AB}_{5} + \underbrace{BC}_{10}$

$\therefore$ AB = 5 $\wedge$ AC = 15

Clave A

8.



M punto medio de $\overline{AB}$:

$\left.\begin{array}{l} x_2 = \dfrac{0+6}{2} = 3 \\ y_2 = \dfrac{0+8}{2} = 4 \end{array}\right\} (x_2; y_2) = (3; 4)$

N punto medio de $\overline{MP}$:

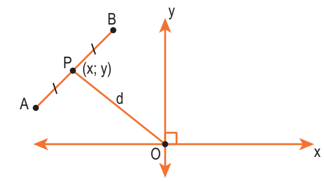
$\left.\begin{array}{l} x_1 = \dfrac{x_2+11}{2} = \dfrac{3+11}{2} = 7 \\ y_1 = \dfrac{y_2+4}{2} = \dfrac{4+4}{2} = 4 \end{array}\right\} (x_1; y_1) = (7; 4)$

$\Rightarrow x = \sqrt{(6-7)^2+(8-4)^2} = \sqrt{1^2+4^2} = \sqrt{17}$

$\therefore x = \sqrt{17}$

Clave A

9.



P es punto medio de $\overline{AB}$.

Los extremos (–9; 2) y (–3; 10).

Luego:

$x = \dfrac{-9+(-3)}{2} = -6$

$y = \dfrac{2+10}{2} = 6$

$\Rightarrow (x; y) = (-6; 6)$

Luego: $\overline{OP}$ es radio vector.

$\Rightarrow d^2 = x^2 + y^2$

$d^2 = (-6)^2 + (6)^2 = 72$

$\therefore d = 6\sqrt{2} = 8{,}5$ (aprox.)

Clave B

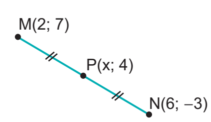
10.

$$\begin{array}{c|cc|c} & -2 & 2 & \\ 4 & 2 & 3 & -6 \\ -3 & -1 & -3 & -6 \\ 6 & -2 & 2 & -2 \\ \hline 7 & & & -14 \end{array}$$

$S = \dfrac{|-14-7|}{2} = \dfrac{|-21|}{2}$ $\qquad \therefore S = 10{,}5$

Clave A

11.



- $x = \dfrac{2+6}{2} \Rightarrow x = 4$
- $y = \dfrac{7+(-3)}{2} \Rightarrow y = 2$

$\therefore x + y = 6$

Clave B

12. • $r = \sqrt{2^2+(-2)^2} = \sqrt{4+4}$

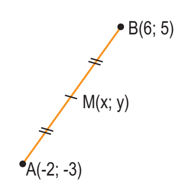
$r = 2\sqrt{2}$

• $A_\bullet = \pi r^2 \Rightarrow A_\bullet = \pi(2\sqrt{2})^2$

$\therefore A_\bullet = 8\pi$

Clave D

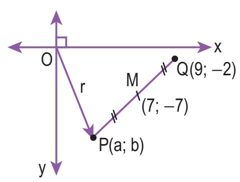
13.



- $x = \dfrac{-2+6}{2} = \dfrac{4}{2} = 2$
- $y = \dfrac{-3+5}{2} = \dfrac{2}{2} = 1$
- $x - y = 2 - 1 = 1$

Clave E

14.



- $7 = \dfrac{a+9}{2} \Rightarrow 14 = a + 9$
  $a = 5$
- $-7 = \dfrac{b-2}{2} \Rightarrow -14 = b - 2$
  $b = -12$
- $r = \sqrt{5^2 + (-12)^2}$
  $r = \sqrt{25+144} = \sqrt{169}$
  $\therefore r = 13$

Clave D

PRACTIQUEMOS

Nivel 1 (página 63) Unidad 4

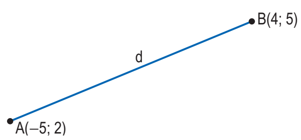
Comunicación matemática

1.

2.

Razonamiento y demostración

3.



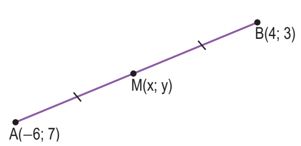
$d = \sqrt{(4-(-5))^2 + (5-2)^2}$

$d = \sqrt{9^2 + 3^2} = \sqrt{90}$

$\therefore d = 3\sqrt{10}$

Clave B

4.



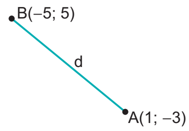
$x = \dfrac{4+(-6)}{2} = -1$

$y = \dfrac{3+7}{2} = 5$

$\therefore$ M(−1; 5)

Clave A

5.



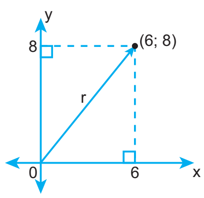
$d = \sqrt{(-5-1)^2 + (5-(-3))^2}$

$d = \sqrt{(-6)^2 + 8^2} = \sqrt{100}$

$\therefore\ d = 10$

Clave E

6.



r es radio vector:

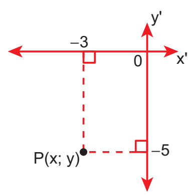
$\Rightarrow r^2 = 6^2 + 8^2$

$r^2 = 100$

$\therefore\ r = 10$

Clave E

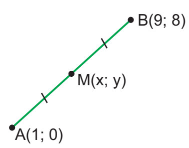
7.



Del gráfico:
$x = -3$
$y = -5$
$\therefore P(x; y) = P(-3; -5)$

Clave B

8.



Por propiedad:

$x = \dfrac{9+1}{2} = 5$

$y = \dfrac{8+0}{2} = 4 \quad \therefore\ M\ (5; 4)$

Clave D

9.

Empleando la distancia entre dos puntos:

$5 = \sqrt{(6-3)^2 + (y-(-2))^2}$

$25 = 9 + (y+2)^2$

$\Rightarrow (y+2)^2 = 16 \Rightarrow |y+2| = 4$

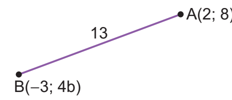
$\Rightarrow y + 2 = 4 \quad \vee \quad y + 2 = -4$

$y = 2 \qquad\qquad y = -6$

$\therefore y = -6 \vee y = 2$

Clave E

10. Por dato:



Empleando la distancia entre dos puntos:

$13 = \sqrt{(-3-2)^2 + (4b-8)^2}$

$169 = 25 + (4b-8)^2$

$144 = (4b-8)^2$

$\Rightarrow |4b-8| = 12$

$\Rightarrow 4b - 8 = 12 \ \vee\ 4b - 8 = -12$

$b = 5 \qquad\qquad b = -1$

$\therefore\ b = 5 \vee b = -1$

Clave A

Resolución de problemas

11. $d_{BC} = \sqrt{(2-\sqrt{x})^2 + (4-(-3))^2} = 5\sqrt{2}$

$\Rightarrow\ (2-\sqrt{x})^2 + 49 = 50$

$2 - \sqrt{x} = \pm 1$ pero $x > 5$

$\Rightarrow\ 2 - \sqrt{x} = -1\ \therefore\ x = 9$

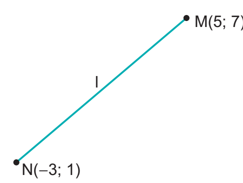
$d_{AC} = \sqrt{(-4-3)^2 + (-2-(-3))^2}$

$d_{AC} = \sqrt{49+1} = \sqrt{50}$

$\therefore\ d_{AC} = 5\sqrt{2}$

Clave C

12.



$L = \sqrt{(5-(-3))^2 + (7-1)^2}$

$L = \sqrt{64+36} = \sqrt{100} = 10$

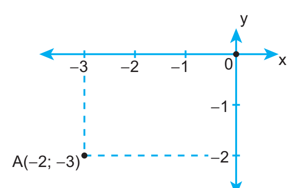
$\therefore$ Perímetro $= 4L = 40$

Clave B

Nivel 2 (página 64) Unidad 4

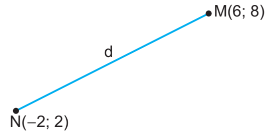
Comunicación matemática

13. I. F



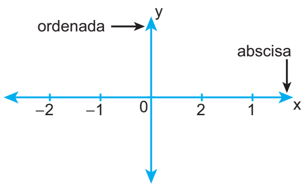
$A \in$ IIIC

II. V



$$d = \sqrt{(6-(-2))^2 + (8-2)^2}$$
$$d = \sqrt{8^2 + 6^2}$$
$$d = \sqrt{100} = 10$$

III. F



- Para que se ubique sobre el eje y su abscisa tiene que ser igual a cero.

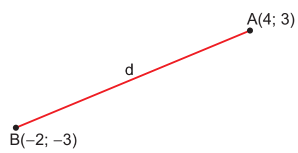
Clave B

14. Para determinar a qué cuadrante pertenece cada punto, solo tendremos en cuenta el signo de las abscisas y ordenadas.

$M(-\,;+) \in$ IIC
$N(-\,;-) \in$ IIIC
$O(+\,;-) \in$ IVC
$P(+\,;+) \in$ IC
$Q(-\,;+) \in$ IIC
$K(+;+) \in$ IC
$S(-\,;+) \in$ IIC
$T(+\,;+) \in$ IC

Razonamiento y demostración

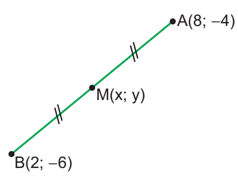
15.



$$d = \sqrt{(4-(-2))^2 + (3-(-3))^2}$$
$$d = \sqrt{6^2 + 6^2} = \sqrt{72} = \sqrt{36.2}$$
$$\therefore d = 6\sqrt{2}$$

Clave D

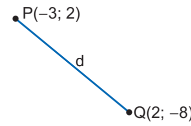
16.



$$x = \frac{8+2}{2} = 5$$
$$y = \frac{-4+(-6)}{2} = -5 \quad \therefore M(5; -5)$$

Clave C

17.



$$d = \sqrt{(-3-2)^2 + (2-(-8))^2}$$
$$d = \sqrt{(-5)^2 + 10^2} = \sqrt{125}$$
$$\therefore d = 5\sqrt{5}$$

Clave A

18. Del gráfico:

$x = -4$
$y = 3$
$\therefore\ P(x; y) = P(-4; 3)$

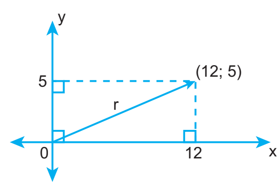
Clave A

19. Del gráfico:

$x = \sqrt{5}$
$y = \sqrt{3}$
$\therefore\ Q(x; y) = Q(\sqrt{5}; \sqrt{3})$

Clave E

20.

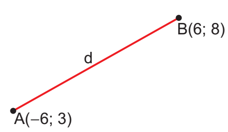


r es radio vector:

$$\Rightarrow r^2 = 12^2 + 5^2 = 144 + 25$$
$$r^2 = 169$$
$$\therefore\ r = 13$$

Clave B

21.



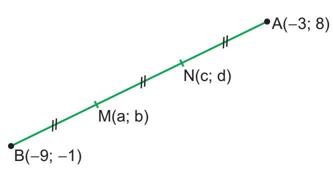
Empleando la distancia entre dos puntos:

$$d = \sqrt{(-6-6)^2 + (3-8)^2}$$
$$d = \sqrt{(-12)^2 + (-5)^2} = \sqrt{169}$$
$$\therefore d = 13$$

Clave B

Resolución de problemas

22.



En el segmento BN:

$a = \dfrac{-9+c}{2} \Rightarrow 2a = -9 + c$ ... (I)

$b = \dfrac{-1+d}{2} \Rightarrow 2b = -1 + d$ ... (II)

En el segmento MA:

$c = \dfrac{a-3}{2} \Rightarrow 2c = a - 3$ ... (III)

$d = \dfrac{b+8}{2} \Rightarrow 2d = b + 8$ ... (IV)

De (I) ∧ (III):

$2(2c + 3) = -9 + c$
$3c = -15 \Rightarrow c = -5$
$a = -7$

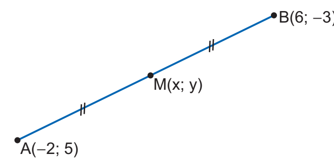
De (II) ∧ (IV)

$2(2d - 8) = -1 + d$
$3d = 15 \Rightarrow d = 5$
$b = 2$

$\Rightarrow (a + d) - (b + c) = (-7 + 5) - (2 - 5)$
$= -2 - (-3)$
$\therefore (a + d) - (b + c) = 1$

Clave E

23.



$$x = \frac{6+(-2)}{2} = 2$$
$$y = \frac{5+(-3)}{2} = 1$$
$$\therefore\ x + y = 2 + 1 = 3$$

Clave C

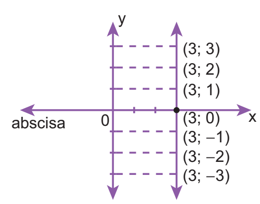
Nivel 3 (página 65) Unidad 4

Comunicación matemática

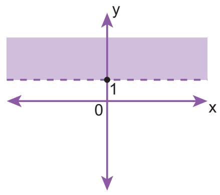
24.
- $Q(-3; 5) \in$ IIC F
- $d = \sqrt{(3-0)^2 + (6-2)^2}$
  $d = \sqrt{3^2 + 4^2} = 5$ V
- $x = \dfrac{-3+1}{2} = -1$
  $y = \dfrac{6-2}{2} = 2;\ M = (-1; 2)$ V
- $R(4; 6) \in$ IC F
  $S(2; -3) \in$ IVC

25.

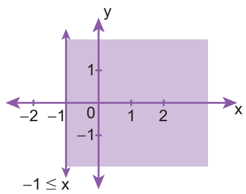
a) Unimos todos los puntos con abscisa 3 y obtenemos una recta vertical.



b) En este caso $y > 1$

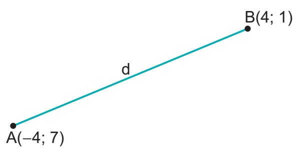


c)



Razonamiento y demostración

26.



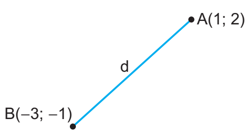
$d = \sqrt{(4-(-4))^2+(1-7)^2}$

$d = \sqrt{8^2+(-6)^2} = \sqrt{100}$

$\therefore\ d = 10$

Clave B

27.



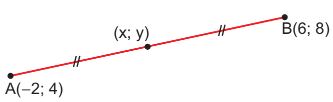
$d = \sqrt{(1-(-3))^2+(2-(-1))^2}$

$d = \sqrt{4^2+3^2} = \sqrt{25}$

$\therefore\ d = 5$

Clave C

28.



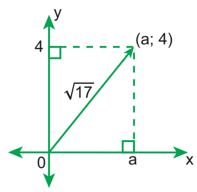
$x = \dfrac{6+(-2)}{2} = 2 \Rightarrow x = 2$

$y = \dfrac{8+4}{2} = 6 \Rightarrow y = 6$

$\therefore\ (x + y) = 2 + 6 = 8$

Clave D

29.



Usando la propiedad del radio vector:

$(\sqrt{17})^2 = (a)^2 + (4)^2$

$17 = a^2 + 16 \Rightarrow a^2 = 1$

$\Rightarrow a^2 - 1 = 0$

$(a + 1)(a - 1) = 0$

$a = -1 \ \vee\ a = 1$

Según el gráfico $a > 0 \Rightarrow a = 1$

Clave E

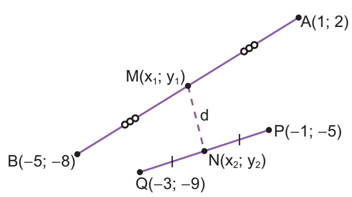
30.

$d = \sqrt{(-3-2)^2+(-2-6)^2}$

$d = \sqrt{5^2+8^2} = \sqrt{89} \approx 9{,}4$

Clave C

31.



$M(x_1; y_1)$ es punto medio de $\overline{AB}$.

$\Rightarrow x_1 = \dfrac{(-5)+(1)}{2} \Rightarrow x_1 = -2$

$\Rightarrow y_1 = \dfrac{(-8)+(2)}{2} \Rightarrow y_1 = -3$

$\Rightarrow M(x_1; y_1) = M(-2; -3)$

$N(x_2; y_2)$ es punto medio de $\overline{PQ}$.

$\Rightarrow x_2 = \dfrac{(-3)+(-1)}{2} \Rightarrow x_2 = -2$

$\Rightarrow y_2 = \dfrac{(-9)+(-5)}{2} \Rightarrow y_2 = -7$

$\Rightarrow N(x_2; y_2) = N(-2; -7)$

Piden:

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d = \sqrt{(-2-(-2))^2+(-7-(-3))^2}$

$d = \sqrt{(-2+2)^2+(-7+3)^2} = \sqrt{16}$

$\therefore d = 4$

Clave D

Resolución de problemas

32. De la figura:

$r^2 = x^2 + 2$

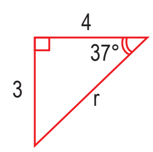
$11 = x^2 + 2$

$\Rightarrow x^2 = \pm 3$; como $x \in$ IIC

$\Rightarrow x = -3$

Clave C

33. De la figura:



$\Rightarrow r = 5$

$y = -3$

$\therefore r + y = 2$

Clave E

34. Como son consecutivos hallamos el lado (L):

$L = \sqrt{(m+2-(m-2))^2+n-3-(n+1))^2}$

$L = \sqrt{(2+2)^2+(-3-1)^2} = \sqrt{4^2+4^2}$

$L = 4\sqrt{2}$

$\therefore$ Perímetro $= 4L = 16\sqrt{2}$

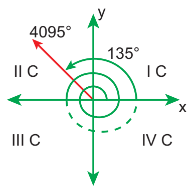
Clave D

RAZONES TRIGONOMÉTRICAS DE UN ÁNGULO EN POSICIÓN NORMAL

APLICAMOS LO APRENDIDO (página 66) Unidad 4

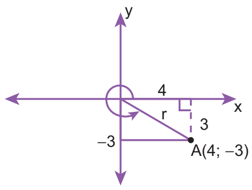
1. Dividimos entre 360°, el resto nos indicará el cuadrante al que pertenece.

$$\begin{array}{r|l} 4095° & 360° \\ \hline \underline{360°} & 11 \\ 495° & \\ \underline{360°} & \\ 135° & \end{array}$$



Clave A

2. Graficamos el ángulo α:



Hallamos el radio vector:

$r = \sqrt{x^2 + y^2} \Rightarrow r = \sqrt{4^2 + (-3)^2}$

$r = \sqrt{25}$

$\therefore\ r = 5$

Nos piden:

$F = \sec\alpha\tan\alpha$

$F = \frac{r}{x}.\frac{y}{x}$

$F = \frac{5}{4}.-\frac{3}{4} = -\frac{15}{16}$

Clave C

3. Hallamos el senα:

$32\text{sen}^5\alpha = -1$

$\text{sen}^5\alpha = -\frac{1}{32}$

$\text{sen}\alpha = \sqrt[5]{-\frac{1}{32}} \Rightarrow \text{sen}\alpha = -\frac{1}{2} = \frac{y}{r}$

Por radio vector, sabemos:

$x^2 + y^2 = r^2$

$x^2 + (-1)^2 = (2)^2$

$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

Pero $\cos\alpha < 0$:

$\Rightarrow x = -\sqrt{3} \wedge \cos\alpha = -\frac{\sqrt{3}}{2}$

$\tan\alpha = \frac{\sqrt{3}}{3}$

Reemplazamos en M:

$M = \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)}{\frac{\sqrt{3}}{3}}$ $\quad\therefore\ M = 3/4$

Clave D

4. Por dato: $\cos\frac{x}{2} = -\frac{1}{5}$

Piden: cosx

Sabemos: $\cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$

Entonces deducimos que:

$-\frac{1}{5} = -\sqrt{\frac{1 + \cos x}{2}}$

$\left(-\frac{1}{5}\right)^2 = \left(-\sqrt{\frac{1 + \cos x}{2}}\right)^2$

$\frac{1}{25} = \frac{1 + \cos x}{2}$

$2 = 25 + 25\cos x$

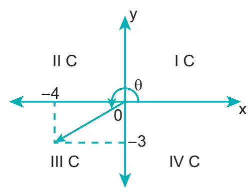
$\Rightarrow 25\cos x = -23$

$\therefore\ \cos x = -\frac{23}{25}$

Clave B

5. Hallamos un punto final del ángulo:

$\theta \in$ IIIC:



$x^2 + y^2 = r^2$

$(-4)^2 + (-3)^2 = r^2 \Rightarrow r = 5$

$\sec\theta = r/x \ \wedge\ \csc\theta = r/y$

$\sec\theta = -5/4 \ \wedge\ \csc\theta = -5/3$

Reemplazamos en k:

$k = \sec\theta\csc\theta$

$k = \left(-\frac{5}{4}\right)\left(-\frac{5}{3}\right) = \frac{25}{12}$

Clave B

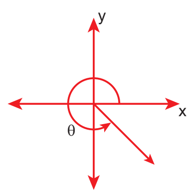
6. $\tan^3\theta + 30 = 3;\ \csc\theta < 0$

$\tan^3\theta = 3 - 30$

$\tan^3\theta = -27$

$\tan\theta = \sqrt[3]{-27}$

$\tan\theta = -3 \wedge \csc\theta < 0 \Rightarrow \theta \in$ IV C



En forma práctica: $x = 1,\ y = -3 \Rightarrow r = \sqrt{10}$

Piden: $Q = \text{sen}\theta\ .\ \cos\theta\ .\ \tan\theta$

$Q = \left(\frac{y}{r}\right)\left(\frac{x}{r}\right)\left(\frac{y}{x}\right) = \frac{y^2}{r^2} = \frac{9}{10}$

$\therefore\ Q = \frac{9}{10}$

Clave B

7. $\quad$ IIC $\quad$ IIIC $\quad$ IVC

$K = \text{sen}125°\ .\ \tan185°\ .\ \cos355°$

$K = \underbrace{(+)(+)}(+)$

$K = \quad (+)(+) \qquad \therefore\ K = (+)$

Clave A

8. Hallamos el radio vector:

$x^2 + y^2 = r^2$

$(-2)^2 + (1)^2 = r^2 \Rightarrow r = \sqrt{5}$

$\text{sen}^2\alpha = \left(\frac{y}{r}\right)^2 = \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{1}{5}$

$\tan^2\alpha = \left(\frac{y}{x}\right)^2 = \left(\frac{1}{-2}\right)^2 = \frac{1}{4}$

Reemplazamos en P:

$\therefore\ P = (1/5) + (1/4) = 9/20$

Clave C

9. Sean α y β los ángulos $\alpha < \beta$

$\alpha/\beta = 1/3 \Rightarrow \alpha = k$

$\beta = 3k$

Además:

$\beta - \alpha = 360°\ n$

$3k - k = 360°\ n$ (definición)

$2k = 360°\ n \Rightarrow k = 180°\ n$

Si:

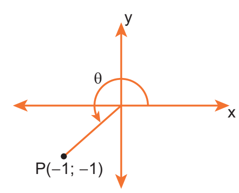
$n = 1 \Rightarrow k = 180° \Rightarrow \alpha = 180° \wedge \beta = 540°$

$n = 2 \Rightarrow k = 360° \Rightarrow \alpha = 360° \wedge \beta = 1080°$

$n = 3 \Rightarrow k = 540° \Rightarrow \alpha = 540° \wedge \beta = 1620°$

Clave D

10.



$x = -1,\ y = -1 \Rightarrow r = \sqrt{2}$

$\text{sen}\theta = \frac{y}{r} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$

$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x} = \frac{-1}{-1} = 1$

Piden: $N = \text{sen}\theta + \cos\theta - \tan\theta$

$N = \left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right) - (1)$

$\therefore N = -\sqrt{2} - 1$

Clave D

11. $P = \dfrac{\overbrace{\tan 185°}^{(+)}\ .\ \overbrace{\text{sen}125°}^{(+)}}{\underbrace{\cos 225°}_{(-)}\ .\ \underbrace{\cot 135°}_{(-)}}$

$P = \frac{(+)(+)}{(-)(-)} = \frac{(+)}{(+)} = (+)$

$\therefore$ P es (+).

Clave D

12. Si β y θ son coterminales, entonces:

$\text{sen}\beta = \text{sen}\theta$

$\tan\beta = \tan\theta$

Reemplazamos en P:

$$P = \frac{\text{sen}^2\theta + \cos^2\theta}{\tan^2\theta + 1}\sec^2\theta$$

$\text{sen}\theta = y/r$; $\cos\theta = x/r$; $\tan\theta = y/x$; $\sec\theta = r/x$

$$\frac{\left(\frac{y^2}{r^2} + \frac{x^2}{r^2}\right)\frac{r^2}{x^2}}{\frac{y^2}{x^2} + 1} = 1$$

Clave D

13. Dato: $45° < x < 86°$ ...(i)

$(i)(\times 2) \Rightarrow 90° < 2x < 172°$

$(2x) \in \text{II C}$

$(i)(\times 3) \Rightarrow 135° < 3x < 258°$

$(3x) \in \text{III C}$

$(i)(\times 4) \Rightarrow 180° < 4x < 344°$

$(4x) \in \text{III C o IV C}$

Piden el signo de:

$$M = \underbrace{\text{sen}2x}_{(+)} . \underbrace{\cos 3x}_{(-)} . \underbrace{\tan 4x}_{(+)\,o\,(-)}$$

Si: $(4x) \in \text{IIIC} \Rightarrow \tan 4x$ es $(+)$

$M = (+)(-)(+)$

$\Rightarrow M = (-)$

Si: $(4x) \in \text{IV C} \Rightarrow \tan 4x$ es $(-)$

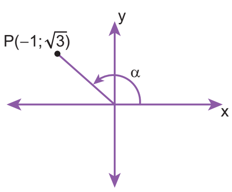
$M = (+)(-)(-)$

$M = (+)$

$\therefore$ M puede tener signo $(-)$ o $(+)$

Clave E

14.



Del gráfico: $x = -1$, $y = \sqrt{3} \Rightarrow r = 2$

$$\tan\alpha = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\cot\alpha = \frac{x}{y} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\text{sen}\alpha = \frac{y}{r} = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Piden:

$M = \tan\alpha + \cot\alpha - \text{sen}\alpha$

$$M = (-\sqrt{3}) + \left(-\frac{\sqrt{3}}{3}\right) - \left(\frac{\sqrt{3}}{2}\right)$$

$$M = -\sqrt{3} - \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{2}$$

$$\therefore M = -\frac{11\sqrt{3}}{6}$$

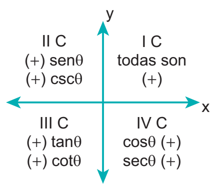
Clave C

PRACTIQUEMOS
Nivel 1 (página 68) Unidad 4

Comunicación matemática

1.

2. Reconocemos el signo de las razones en los cuadrantes:

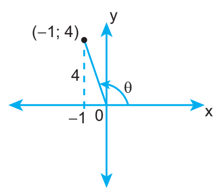


Entonces:

Si $\theta \in \text{IC} \Rightarrow$	$\text{sen}\theta$ es	$(+)$
Si $\theta \in \text{IIIC} \Rightarrow$	$\cos\theta$ es	$(-)$
Si $\theta \in \text{IIC} \Rightarrow$	$\tan\theta$ es	$(-)$
Si $\theta \in \text{IVC} \Rightarrow$	$\sec\theta$ es	$(+)$
Si $\theta \in \text{IIC} \Rightarrow$	$\text{sen}\theta$ es	$(+)$

Razonamiento y demostración

3.

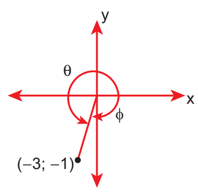


Del gráfico:

$\cot\theta = -1/4$

Clave C

4.



Del gráfico:

$x = -3$, $y = -1 \Rightarrow r = \sqrt{10}$

Piden:

$K = \sec\phi \,.\, \csc\phi = \sec\theta \,.\, \csc\theta$

(ϕ y θ son coterminales)

$$K = \left(\frac{r}{x}\right)\left(\frac{r}{y}\right)$$

$$K = \left(\frac{\sqrt{10}}{-3}\right)\left(\frac{\sqrt{10}}{-1}\right) = \frac{10}{3}$$

$$\therefore K = \frac{10}{3}$$

Clave A

5. $$J = \frac{\text{sen}100° . \cos 200°}{\tan 300°} = \frac{(+).(-)}{(-)} = \frac{(-)}{(-)} = (+)$$

$\therefore$ J es $(+)$

Clave A

6. Dato: $\tan\beta = -3$; $\beta \notin \text{II C} \Rightarrow \beta \in \text{IV C}$

Usamos el dato: $\beta \in \text{IV C} \Rightarrow x > 0 \wedge y < 0$

$$\tan\beta = \frac{-3}{1} = \frac{y}{x}$$

En forma práctica:

$y = -3$, $x = 1 \Rightarrow r = \sqrt{10}$

Piden: $J = \sec\beta + \csc\beta$

$$J = \left(\frac{r}{x}\right) + \left(\frac{r}{y}\right) = \left(\frac{\sqrt{10}}{1}\right) + \left(\frac{\sqrt{10}}{-3}\right)$$

$$\therefore J = \frac{2\sqrt{10}}{3}$$

Clave B

7. $\alpha \in \text{II C} \Rightarrow x < 0 \wedge y > 0$

$\tan\alpha = -\sqrt{3}$

$$\tan\alpha = \frac{\sqrt{3}}{-1} = \frac{y}{x} \Rightarrow x = -1, y = \sqrt{3} \Rightarrow r = 2$$

Piden:

$$\csc\alpha = \frac{r}{y} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\therefore \csc\alpha = \frac{2\sqrt{3}}{3}$$

Clave A

8. $\text{sen}\beta : \cos\beta < 0$; $|\text{sen}\beta| = -\text{sen}\beta$

$\text{sen}\beta \,.\, \cos\beta < 0$

$(+) . (-) \Rightarrow \beta \in \text{II C}$ (opción 1)

$(-) . (+) \Rightarrow \beta \in \text{IV C}$ (opción 2)

$|\text{sen}\beta| = -\text{sen}\beta \Rightarrow \text{sen}\beta < 0$

La opción 2 cumple con las condiciones.

$\therefore \beta \in \text{IV C}$

Clave D

Resolución de problemas

9. $\alpha + \theta = 180°$

$\alpha - \theta = 360°$

$2\alpha = 540°$

$\alpha = 270°$

$\Rightarrow \theta = -90°$

Piden:

$$M = \frac{\text{sen}\alpha - \cos\theta}{\text{sen}\theta}$$

$$M = \frac{\text{sen}270° - \cos(-90°)}{\text{sen}(-90°)}$$

$$\Rightarrow M = \frac{(-1)-(0)}{-1} = \frac{-1}{-1} = 1$$

$\therefore M = 1$

Clave A

10. Del gráfico hallamos las coordenadas de M:

$M = (-a; -a/2)$

Como θ está en posición normal, entonces:

$$\tan\theta = y/x = \frac{-a/2}{-a} = \frac{1}{2}$$

Clave B

Nivel 2 (página 68) Unidad 4

Comunicación matemática

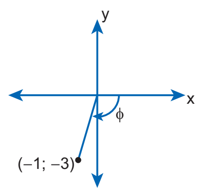
11. Hallamos el número de vueltas:

$792° - 72° = 360°(2)$ (V)
$446° - 86° = 360°(1)$ (V)
$1280° - 72° = 360°(3) + 200°$ (F)
$2260° - 160° = 360°(6) + 100°$ (F)
$1972° - 272° = 360°(5) + 172°$ (F)
$820° - 100° = 360°(2)$ (V)

12.

Razonamiento y demostración

13.



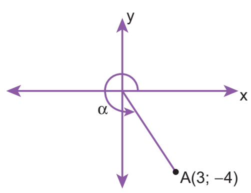
Del gráfico:
$x = -1,\ y = -3$

Piden: $\tan\phi = \frac{y}{x} = \frac{-3}{-1}$

$\therefore \tan\phi = 3$

Clave C

14.



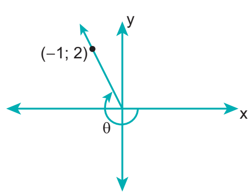
En el gráfico:
$x = 3,\ y = -4 \Rightarrow r = 5$

Piden: $\operatorname{sen}\alpha = \frac{y}{r} = \frac{-4}{5}$

$\therefore \operatorname{sen}\alpha = -\frac{4}{5} = -0{,}8$

Clave E

15.



En el gráfico:
$x = -1,\ y = 2 \Rightarrow r = \sqrt{5}$

Piden:
$K = \operatorname{sen}\theta \cdot \cos\theta$

$K = \left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$

$K = \left(\frac{2}{\sqrt{5}}\right)\left(\frac{-1}{\sqrt{5}}\right)$

$K = -\frac{2}{5} = -0{,}4$

$\therefore K = -0{,}4$

Clave C

16. $J = \dfrac{\cos 100° - \operatorname{sen}140°}{\tan 120° + \cot 300°} = \dfrac{(-)-(+)}{(-)+(-)} = \dfrac{(-)}{(-)}$

$J = \dfrac{(-)}{(-)} = (+)$

$\therefore$ J es $(+)$.

Clave A

17. $\operatorname{sen}\alpha = \frac{2}{3};\ \alpha \in \text{IIC}$

$\operatorname{sen}\alpha = \frac{2}{3} = \frac{y}{r}$

En forma práctica:
$y = 2,\ r = 3 \Rightarrow \underbrace{x = -\sqrt{5}}_{\alpha \in \text{IIC}}$

Piden:
$J = \sec\alpha \cdot \csc\alpha$

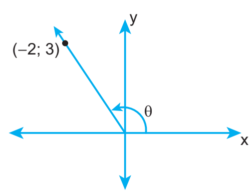
$J = \left(\frac{r}{x}\right)\cdot\left(\frac{r}{y}\right)$

$J = \left(\frac{3}{-\sqrt{5}}\right)\left(\frac{3}{2}\right) = \frac{9}{-2\sqrt{5}} = -\frac{9\sqrt{5}}{10}$

$\therefore J = -\frac{9\sqrt{5}}{10} = -0{,}9\sqrt{5}$

Clave B

18.



Del gráfico:
$x = -2,\ y = 3 \Rightarrow r = \sqrt{13}$

Piden:
$K = \sec\theta + \csc\theta$

$K = \left(\frac{r}{x}\right) + \left(\frac{r}{y}\right)$

$K = \left(\frac{\sqrt{13}}{-2}\right) + \left(\frac{\sqrt{13}}{3}\right) = \frac{3\sqrt{13} - 2\sqrt{13}}{-6} = \frac{\sqrt{13}}{-6}$

$\therefore K = -\frac{\sqrt{13}}{6}$

Clave D

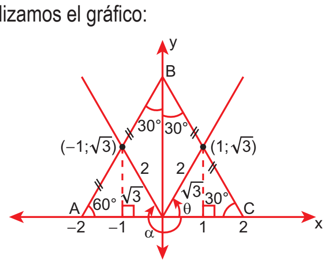
Resolución de problemas

19. Reemplazamos x por 180°.

$F(180°) = \operatorname{sen}180° - \cos 360° + \csc 90°$
$F(180°) = 0 - 1 + 1$
$F(180°) = 0$

Clave E

20. Analizamos el gráfico:



Luego:
$T = \operatorname{sen}\alpha\cos\theta$

$T = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4}$

Clave B

Nivel 3 (página 69) Unidad 4

Comunicación matemática

21. $M = \dfrac{(\cos 0° + \operatorname{sen}90°)^2}{\cos 180°}\csc 270°$

$M = \dfrac{(1+1)^2}{-1}(-1) \Rightarrow M = 4$

$N = (\tan 180° - \cos 360°)(\sec 180° + \csc 270°)$
$N = (0 - 1)(-1 + (-1))$
$N = (-1)(-2) \Rightarrow N = 2$
$\therefore 2N = M$

Clave D

22. I. Si $\alpha = 180°$
$k = \operatorname{sen}90°\cot 360°$
$k = (1) \cdot (\text{ND})$
$k = (\text{ND})$ (F)

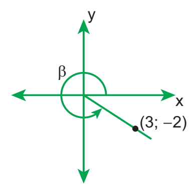
II. Si $\alpha = -180°$
$k = \operatorname{sen}(-90°)\cot(-360°)$
$k = (-1) \cdot (\text{ND})$
$k = (\text{ND})$ (F)

III. Si $\alpha = 630°$
$k = \operatorname{sen}(315°)\cot(1260°)$
$k = (-1) \cdot (\text{ND})$
$k = \text{ND}$ (F)

Clave E

Razonamiento y demostración

23.

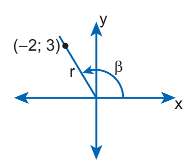


$\tan\beta = \frac{y}{x} = \frac{-2}{3}$

$\therefore \tan\beta = -\frac{2}{3}$

Clave B

24.



$r^2 = (-2)^2 + 3^2$
$r^2 = 4 + 9 = 13$
$r = \sqrt{13}$

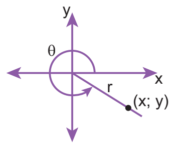
$\operatorname{sen}\beta = \frac{y}{r} = \frac{3}{\sqrt{13}}$

$\cos\beta = \frac{x}{r} = \frac{-2}{\sqrt{13}} = -\frac{2}{\sqrt{13}}$

Piden:
$T = \operatorname{sen}\beta \cdot \cos\beta = \left(\frac{3}{\sqrt{13}}\right)\left(-\frac{2}{\sqrt{13}}\right) = -\frac{6}{13}$

Clave D

25.



$\tan\theta = \dfrac{-1}{\sqrt{6}} = \dfrac{y}{x}$

$x = \sqrt{6},\ \ y = -1 \Rightarrow\ r = \sqrt{7}$

$\text{sen}\theta = \dfrac{y}{r} = \dfrac{-1}{\sqrt{7}} = -\dfrac{1}{\sqrt{7}}$

$\cos\theta = \dfrac{x}{r} = \dfrac{\sqrt{6}}{\sqrt{7}}$

Piden:

$S = \text{sen}\theta\ .\ \cos\theta = \left(-\dfrac{1}{\sqrt{7}}\right)\left(\dfrac{\sqrt{6}}{\sqrt{7}}\right) = -\dfrac{\sqrt{6}}{7}$

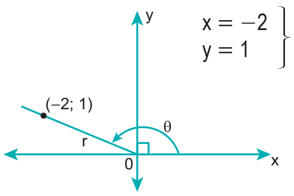
Clave B

26. $\underbrace{\text{sen}\alpha > 0}_{\alpha \in \text{IC} \vee \text{IIC}} \quad \wedge \quad \underbrace{\cos\alpha < 0}_{\alpha \in \text{IIC} \vee \text{IIIC}}$

De ambas condiciones:
$\alpha \in \text{IIC}$

Clave B

27.



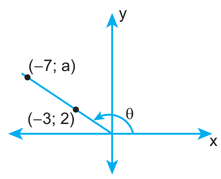
$\left.\begin{matrix} x = -2 \\ y = 1 \end{matrix}\right\}\ r = \sqrt{5}$

Piden:

$\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{5}}{-2} = -\dfrac{\sqrt{5}}{2}$

Clave B

28.



$\tan\theta = \dfrac{y}{x}$

$\tan\theta = \dfrac{2}{-3}$ ...(I)

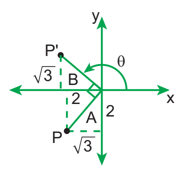
$\tan\theta = \dfrac{a}{-7}$ ...(II)

De (I) y (II): $\dfrac{2}{-3} = \dfrac{a}{-7} \Rightarrow a = \dfrac{14}{3}$

Clave C

Resolución de problemas

29.



Al rotar P 90° podemos observar los triángulos rectángulos simétricos, el punto P' es la nueva ubicación de P: $(-2;\ \sqrt{3})$

$\Rightarrow \cos\theta = \dfrac{-2}{r};\ r^2 = 3 + 4 = 7$

$\Rightarrow r = \sqrt{7}$

$\therefore \cos\theta = \dfrac{-2}{\sqrt{7}} = -\dfrac{2\sqrt{7}}{7}$

Clave A

30. $\sec\alpha = \dfrac{r}{1};$ $\qquad \tan\beta = \dfrac{\sqrt{7}}{2}$

$r^2 = 4 + 1 = 5$

$\Rightarrow r = \sqrt{5}$

Piden: $\dfrac{r^2}{1} + \dfrac{\sqrt{7}^2}{2^2} = 5 + \dfrac{7}{4} = \dfrac{27}{4}$

Clave C

31. Sabemos que $\cot = \dfrac{x_1}{y_1}$

$\Rightarrow \dfrac{2m}{-6} = \dfrac{-4}{\frac{4}{3}m}$

$m^2 = 9$

$m = \pm 3$ con $\alpha \in \text{IIIC}$.

$\Rightarrow m = -3$

Clave D

32. Sean los ángulos α y β, $\alpha < \beta$

$\dfrac{\alpha}{\beta} = \dfrac{3}{4} = k \Rightarrow \begin{matrix} \alpha = 3k \\ \beta = 4k \end{matrix}$

$\beta - \alpha = 360°\ n$

$4k - 3k = 360°\ n$

$k = 360°\ n$

Si $n = 1 \Rightarrow k = 360° \Rightarrow \alpha = 1080° \wedge \beta = 1440°$

Si $n = 2 \Rightarrow k = 720° \Rightarrow \alpha = 2160°$

$\beta = 2880°$

$\therefore \alpha + \beta = 2520°$

Clave C

33. $\text{sen}\theta = y/r = -4/5$

$\Rightarrow\ y = -4\ \wedge\ r = 5$

- $x^2 + y^2 = r^2$

  $x^2 = 5^2 - 4^2\ \Rightarrow\ x = \pm 3$

$\Rightarrow \tan\theta = \dfrac{4}{3}\ \vee\ \tan\theta = \dfrac{-4}{3}$

También:

$\tan\theta = \dfrac{5-3a}{2a-3} = \dfrac{4}{3}\ \vee \tan\theta = \dfrac{5-3a}{2a-3} = \dfrac{-4}{3}$

$15 - 9a = 8a - 12\ \vee\ 15 - 9a = -8a + 12$

$27 = 17a \qquad\qquad a = -3$

$\dfrac{27}{17} = a$

Como $a \in \mathbb{Z}$:

$\therefore a = 3\ \wedge\ \theta \in \text{IVC}$

Clave E

REDUCCIÓN AL PRIMER CUADRANTE

APLICAMOS LO APRENDIDO (página 71) Unidad 4

1. $\text{sen}103° = \text{sen}(180° - 77°) = \text{sen}77°$

$\therefore\ \text{sen}103° = \text{sen}77°$

Clave B

2. $\text{sen}(-300°) = -\text{sen}300°$

$\text{sen}(-300°) = -\text{sen}(360° - 60°)$

$\text{sen}(-300°) = -(-\text{sen}60°)$

$\Rightarrow \text{sen}(-300°) = \text{sen}60° = \dfrac{\sqrt{3}}{2}$

$\therefore \text{sen}(-300°) = \dfrac{\sqrt{3}}{2}$

Clave B

3. $P = \text{sen}(-45°) + \cos(-60°)$

$P = -\text{sen}45° + \cos60°$

$P = -\dfrac{\sqrt{2}}{2} + \dfrac{1}{2} = \dfrac{1-\sqrt{2}}{2}$

$\therefore P = \dfrac{1-\sqrt{2}}{2}$

Clave D

4. $H = \dfrac{2+\tan(-53°)}{\csc(-37°)}$

$H = \dfrac{2+(-\tan 53°)}{-\csc 37°}$

$H = \dfrac{2-\left(\frac{4}{3}\right)}{-\left(\frac{5}{3}\right)} = \dfrac{\frac{2}{3}}{-\frac{5}{3}} = -\dfrac{6}{15} = -\dfrac{2}{5}$

$H = -\dfrac{2}{5} = -0{,}4$

Clave C

5. $P = \text{sen}135° + \cos225° + \sec315°$

$P = (\text{sen}45°) + (-\cos45°) + (\sec45°)$

$P = \text{sen}45° - \cos45° + \sec45°$

$P = \left(\dfrac{\sqrt{2}}{2}\right) - \left(\dfrac{\sqrt{2}}{2}\right) + (\sqrt{2}) = \sqrt{2}$

$\therefore P = \sqrt{2}$

Clave E

6. $\tan2040° - \tan2460°$

$= \tan(360° \,.\, 5 + 240°) - \tan(360° \,.\, 6 + 300°)$

$= \tan240° - \tan300°$

$= \underbrace{\tan(180° + 60°)}_{(\tan60°)} - \underbrace{\tan(360° - 60°)}_{(-\tan60°)}$

$= \tan60° + \tan60° = 2\tan60°$

$= 2(\sqrt{3})$

$\therefore \tan2040° - \tan2460° = 2\sqrt{3}$

Clave B

7. $E = \dfrac{\text{sen}(180°+x)\sec(90°+x)}{\cot(270°+x)}$

$E = \dfrac{(-\text{sen}x)(-\csc x)}{(-\tan x)} = \dfrac{\overbrace{\text{sen}x\csc x}^{1}}{-\tan x}$

$E = \dfrac{1}{-\tan x} = -\cot x$

$\therefore E = -\cot x$

Clave B

8. $E = \text{sen}(360° + \beta) + \cos(270° - \beta)$

$E = (\text{sen}\beta) + (-\text{sen}\beta)$

$E = \text{sen}\beta - \text{sen}\beta = 0$

$\therefore E = 0$

Clave D

9. $M = \text{sen}2940° + \cot3285°$

$2940° \,\lfloor\, 360°$
$2880° \quad 8$
$60°$

$3285° \,\lfloor\, 360°$
$3240° \quad 9$
$45°$

$\text{sen}2940° = \text{sen}60° \ \wedge\ \cot3285° = \cot45°$

$\Rightarrow M = \text{sen}60° + \cot45°$

$M = \left(\dfrac{\sqrt{3}}{2}\right) + 1$

$\therefore M = \dfrac{\sqrt{3}+2}{2}$

Clave C

10. $\csc(-2670°) = -\csc2670°$

$\csc(-2670°) = -\csc(7 \times 360° + 150°)$

$\csc(-2670°) = -\csc150°$

$\csc(-2670°) = -\csc(180° - 30°)$

$\csc(-2670°) = -(\csc30°)$

$\csc(-2670°) = -(2)$

$\therefore \csc(-2670°) = -2$

Clave B

11. $A = -6\sqrt{3}\tan(180° - 60°)$

$A = -6\sqrt{3}(-\tan60°)$

$A = 6\sqrt{3}(\sqrt{3})$

$A = 18$

Clave C

12. $E = \sqrt{4\cos(360° - 60°) + 7}$

$E = \sqrt{4\cos60° + 7}$

$E = \sqrt{4\left(\dfrac{1}{2}\right) + 7}$

$E = 3$

Clave E

13. $S = 6\sqrt{2}\cos(360° \times 1 + 45°)$

$S = 6\sqrt{2}\cos45°$

$S = 6\sqrt{2}\left(\dfrac{\sqrt{2}}{2}\right)$

$S = 6$

Clave A

14. $T = 1 + \sqrt{3}\tan(360° + 240°)$

$T = 1 + \sqrt{3}\tan240°$

$T = 1 + \sqrt{3}\tan(180° + 60°)$

$T = 1 + \sqrt{3}\tan60°$

$T = 1 + \sqrt{3} \,.\, \sqrt{3}$

$T = 4$

Clave D

PRACTIQUEMOS
Nivel 1 (página 73) Unidad 4

Comunicación matemática

1.

2.

Razonamiento y demostración

3. $\text{sen}570° = \text{sen}(360° + 210°) = \text{sen}210°$

$\text{sen}210° = \text{sen}(270° - 60°) = -\cos60°$

$\Rightarrow \text{sen}570° = -\cos60° = -\dfrac{1}{2}$

$\therefore\ \text{sen}570° = -\dfrac{1}{2}$

Clave E

4. $\cot870° = \cot(360°.\,2 + 150°) = \cot150°$

$\cot150° = \cot\underbrace{(180° - 30°)}_{\text{II C}} = -\cot30°$

$\Rightarrow \cot870° = -\cot30° = -(\sqrt{3}) = -\sqrt{3}$

$\therefore\ \cot870° = -\sqrt{3}$

Clave E

5. $\tan750° = \tan(360° \,.\, 2 + 30°)$

$\tan750° = \underbrace{\tan30°}$

$\tan750° = \dfrac{\sqrt{3}}{3}$

$\therefore\ \tan750° = \dfrac{\sqrt{3}}{3}$

Clave A

6. $\cos510° = \cos(360° + 150°) = \cos150°$

$\cos150° = \cos\underbrace{(180° - 30°)}_{\text{II C}} = -\cos30°$

$\Rightarrow \cos510° = -\cos30° = -\left(\dfrac{\sqrt{3}}{2}\right)$

$\therefore\ \cos510° = -\dfrac{\sqrt{3}}{2}$

Clave B

7. $R = 5\sqrt{3} \,.\, \tan600°$

$R = 5\sqrt{3}\,[\tan(360° + 240°)]$

$R = 5\sqrt{3}\tan240°$

$R = 5\sqrt{3}\tan(180° + 60°)$

$R = 5\sqrt{3} \,.\, \tan60° = 5\sqrt{3}(\sqrt{3}) = 15$

$\therefore\ R = 15$

Clave D

8. $A = -2\sqrt{3}\cot150°$

$A = -2\sqrt{3}(-\cot30°)$

$A = 2\sqrt{3}\cot30° = 2\sqrt{3}(\sqrt{3}) = 6$

$\therefore\ A = 6$

Clave A

9. $A = -4\sqrt{3}\tan120°$

$A = -4\sqrt{3}(-\tan60°)$

$A = 4\sqrt{3}\tan60° = 4\sqrt{3}(\sqrt{3}) = 12$

$\therefore\ A = 12$

Clave E

10. $M = 4\sqrt{2}\,(\text{sen}1200°)$

$$\begin{array}{r|l} 1200° & 360° \\ \hline \underline{1080°} & 3 \\ 120° & \end{array}$$

$\Rightarrow \text{sen}1200° = \underbrace{\text{sen}120°}$

$\text{sen}1200° = \text{sen}60°$

$M = 4\sqrt{2}(\text{sen}60°) = 4\sqrt{2}\cdot\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{6}$

$\therefore\ M = 2\sqrt{6}$

Clave B

Nivel 2 (página 73) Unidad 4

Comunicación matemática

11. Por teoría
    - I. V
    - II. V
    - III. V
    - IV. F

12.

Razonamiento y demostración

13. $\text{sen}110° = \text{sen}\underbrace{(180° - 70°)}_{\text{II C}}$

$\text{sen}110° = +\text{sen}70°$

Clave C

14. $M = 3 + 8\text{sen}150°$

$M = 3 + 8(\text{sen}30°)$

$M = 3 + 8\left(\frac{1}{2}\right) = 3 + 4 = 7$

$\therefore\ M = 7$

Clave E

15. $L = 1 - \cot135°$

$L = 1 - (-\cot45°)$

$L = 1 + \cot45°$

$L = 1 + 1 = 2$

$\therefore\ L = 2$

Clave E

16. $S = \text{sen}300° \cdot \cos150°$

$S = (-\text{sen}60°)(-\cos30°) = \text{sen}60° \cdot \cos30°$

$S = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{3}{4}$

$\therefore\ S = \frac{3}{4}$

Clave D

17. $S = \tan300° + 6\cot240°$

$S = (-\tan60°) + 6(\tan30°)$

$S = (-\sqrt{3}) + 6\left(\frac{\sqrt{3}}{3}\right) = -\sqrt{3} + 2\sqrt{3} = \sqrt{3}$

$S = \sqrt{3}$

Clave B

18. $P = \csc150° - 6\text{sen}330°$

$P = (\csc30°) - 6(-\text{sen}30°)$

$P = \csc30° + 6\text{sen}30° = 2 + 6\left(\frac{1}{2}\right) = 5$

$\therefore\ P = 5$

Clave A

19. $R = \sec330° + \sec210°$

$R = (\sec30°) + (-\csc60°)$

$R = \sec30° - \csc60°$

$R = \left(\frac{2\sqrt{3}}{3}\right) - \left(\frac{2\sqrt{3}}{3}\right) = 0$

$\therefore\ R = 0$

Clave D

20. $A = \cos150° - \cos210°$

$A = (-\cos30°) - (-\text{sen}60°)$

$A = -\cos30° + \text{sen}60° = -\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = 0$

$\therefore\ A = 0$

Clave A

Nivel 3 (página 74) Unidad 4

Comunicación matemática

21. Por teoría:
    - I. V
    - II. V
    - III. V

22. Por teoría:
    - I. V
    - II. V
    - III. V

Razonamiento y demostración

23. $V = (6 - 8\cos120°) \cdot \text{sen}150°$

$V = (6 - 8(-\cos60°) \cdot \text{sen}30°$

$V = \left(6 + 8\left(\frac{1}{2}\right)\right)\cdot\left(\frac{1}{2}\right)$

$V = \frac{(6+4)}{2} = \frac{10}{2} = 5$

$\therefore\ V = 5$

Clave B

24. $N = \tan300° - \text{sen}150° + 2\cos210° + \text{sen}30°$

$N = (-\tan60°) - (\text{sen}30°) + 2(-\text{sen}60°) + \text{sen}30°$

$N = -\sqrt{3} - \frac{1}{2} - 2\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}$

$\therefore\ N = -2\sqrt{3}$

Clave D

25. $S = \cos300° \cdot \text{sen}150° + \text{sen}240° \cdot \cos390°$

$S = (\cos60°)(\text{sen}30°) + (-\cos30°)(\cos30°)$

$S = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$S = \frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = -\frac{1}{2} = -0{,}5$

$\therefore\ S = -0{,}5$

Clave C

26. $K = -2\sqrt{2}\sec225° + \csc150°$

$K = -2\sqrt{2}(-\csc45°) + (\csc30°)$

$K = +2\sqrt{2}\csc45° + \csc30°$

$K = 2\sqrt{2}(\sqrt{2}) + (2)$

$K = 4 + 2 = 6$

$\therefore\ K = 6$

Clave E

27. $N = \sqrt{23 - \sec3000°}$

$$\begin{array}{r|l} 3000° & 360° \\ \hline \underline{2880°} & 8 \\ 120° & \end{array}$$

$\Rightarrow \sec3000° = \underbrace{\sec120°}$

$\sec3000° = -\sec60°$

$N = \sqrt{23 - (-\sec60°)} = \sqrt{23 + (2)} = \sqrt{25}$

$\therefore\ N = 5$

Clave B

28. $A = \sqrt{6 - 5(\sec240°)}$

$A = \sqrt{6 - 5(-\csc30°)}$

$A = \sqrt{6 + 5(2)} = \sqrt{6 + 10} = \sqrt{16} = 4$

$\therefore\ A = 4$

Clave A

29. $V = \sqrt{1 - 8\sqrt{2}(\text{sen}225°)}$

$V = \sqrt{1 - 8\sqrt{2}(-\cos45°)}$

$V = \sqrt{1 + 8\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)} = \sqrt{1 + 4(2)}$

$V = \sqrt{9} = 3$

$\therefore\ V = 3$

Clave E

30. $L = \sqrt{3 - 4\sqrt{3}(\cos150°)}$

$L = \sqrt{3 - 4\sqrt{3}(-\cos30°)}$

$L = \sqrt{3 + 4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)} = \sqrt{3 + 2(3)}$

$L = \sqrt{9} = 3$

$\therefore\ L = 3$

Clave A

SISTEMA MÉTRICO DECIMAL

APLICAMOS LO APRENDIDO (página 75) Unidad 4

1. 1 kg ___ 10^5 cg
x ___ 3×10^7 cg

$\Rightarrow x = \dfrac{3.10^7\ \text{cg}.1\ \text{kg}}{10^5\ \text{cg}}$

$\therefore x = 300$ kg

Clave B

2. 1 hm ____ 10^5 mm
x ____ 7×10^9 mm

$\Rightarrow x = \dfrac{1\ \text{hm}.7.10^9\ \text{mm}}{10^5\ \text{mm}}$

$\therefore x = 70\ 000$ hm

Clave C

3. 1 dal _____ 100 dl
80 dal _____ x

$\Rightarrow x = \dfrac{80\ \text{dal}.100\ \text{dl}}{1\ \text{dal}}$

$\therefore x = 8000$ dl

Clave A

4. 1 q _____ 100 kg
x _____ 10^3 kg

$\Rightarrow x = \dfrac{1\text{q}.10^3\ \text{kg}}{100\ \text{kg}}$

$\therefore x = 10$ q

Clave D

5. 1 dam _____ 10^3 cm
x _____ 200 cm

$\Rightarrow x = \dfrac{1\ \text{dam}.200\ \text{cm}}{10^3\ \text{cm}}$

$\therefore x = \dfrac{1}{10}$ dam = 0,2 dam

Clave B

6. 1 kl ____ 10^4 dl
x ____ 10^2 dl

$\Rightarrow x = \dfrac{1\text{kl}.10^2\ \text{dl}}{10^4\ \text{dl}}$

$\therefore x = \dfrac{1}{10^2}$. kl = 0,01 kl

Clave E

7. En A:
1 hg ____ 100 g
x ____ 250 g

$\Rightarrow x = \dfrac{1\ \text{hg}.250\ \text{g}}{100\ \text{g}}$

$\therefore x = 2{,}5$ hg

En B:
1 hg ______ 1000 dg
y ______ 450 dg

$\Rightarrow y = \dfrac{1\ \text{hg}.450\ \text{g}}{1000\ \text{g}}$

$\therefore y = 0{,}45$ hg

En C:
x + y = 2,5 hg + 0,45 hg = 2,95 hg

Clave C

8. $\dfrac{6{,}5.10^3\ \text{hm}}{d}$ a km.

$d = 6{,}5 \ .\ 10^3\ \text{hm} = 6{,}5\ .\ 10^3(0{,}1\ \text{km})$
$= 6{,}5\ .\ 10^2$ km

$M \vdash d/2 \dashv P$

$\dfrac{d}{2} = \dfrac{6{,}5 \times 10^2\text{km}}{2} = \dfrac{650}{2}\text{km} = 325$ km

Clave A

9.
- lu - ma - mi: $3 \times 7{,}5$ hl = 22,5 hl
- ju - vi - sa - do: 4×12 dal = 48 dal = 48 (0,1 hl) = 4,8 hl

Consumo total por semana:
C = 22,5 hl + 4,8 hl = 27,3 hl

1 semana _____ 27,3 hl
x _____ 5,46 kl

$\Rightarrow x = \dfrac{1\ \text{semana}.5{,}46\ \text{kl}}{27{,}3\ \text{hl}}$

$x = 1\ \text{semana}\ 0{,}2\ \dfrac{\text{kl}}{\text{hl}}$

$x = 0{,}2\dfrac{10\ \text{hl}}{\text{hl}}$ semana

$\therefore x = 2$ semanas

Clave D

10. Sea P_t: peso total
$P_t = 15(3{,}6\ \text{q}) + 20(15\ \text{mag})$.
$P_t = 15\ .\ (3{,}6)(10^{-1}\ \text{t}) + 20(15)(10^{-2}\ \text{t})$
$P_t = 5{,}4\ \text{t} + 3\ \text{t} = 8{,}4$ t

Clave C

11. $D_{total} = 1{,}92\ \text{km} = 4k$
$D_{AB} = x = k$

$\Rightarrow x = \dfrac{1{,}92\ \text{km}}{4} = 0{,}48\ \text{km} = 4{,}8$ hm

Clave E

12. Cantidad necesaria: Q_t
$Q_t = 1{,}2\ \text{hl} + 8\ \text{dal} + 250\ \text{cl}$
$Q_t = 1{,}2(10^2\ \text{l}) + 8(10\ \text{l}) + 250(10^{-2}\ \text{l})$
$Q_t = 120\ \text{l} + 80\ \text{l} + 2{,}5\ \text{l}$
$Q_t = 202{,}5$ l

Clave B

13. 1 caja ____ $12\ \overbrace{\text{bolsas}}^{0{,}5\ \text{kg}} = 12 \times (0{,}5\ \text{kg}) = 6$ kg

$\Rightarrow$ n.° cajas $= \dfrac{45\ \text{mag}}{6\ \text{kg}} = \dfrac{45(10\ \text{kg})}{6\ \text{kg}} = 75$

1 bolsa = 0,5 kg

$\Rightarrow$ N.° bolsas $= \dfrac{45\ \text{mag}}{0{,}5\ \text{kg}} = \dfrac{45(10\ \text{kg})}{0{,}5\ \text{kg}} = 900$

Clave A

14.
- 1 dm $\Rightarrow$ S/.5
12 m $\Rightarrow$ x

$\Rightarrow x = \dfrac{12\ \text{m.S/.5}}{1\ \text{dm}} = \dfrac{12(10\ \text{dm}).\text{S/.5}}{1\ \text{dm}}$
x = S/.600

- 1 dam $\Rightarrow$ S/.7
3600 cm $\Rightarrow$ y

$\Rightarrow y = \dfrac{3600\ \text{cm.S/.7}}{1\ \text{dam}} = \dfrac{3600\ \text{cm.S/.7}}{1000\ \text{cm}} = \text{S/.25.2}$

- x + y = S/.600 + S/.25 . 2 = S/.625.2

Clave E

PRACTIQUEMOS
Nivel 1 (página 77) Unidad 4

Comunicación matemática

1.

2.

Razonamiento y demostración

3. 1 t _____ 1000 kg
8 t _____ x
$\Rightarrow x = 1000\ \text{kg} \times 8$
x = 8000 kg

Clave B

4. 1 hg ____ 10^4 cg
x ____ 15×10^4 cg

$\Rightarrow x = \dfrac{15 \times 10^4\ \text{cg} \times 1\ \text{hg}}{10^4\ \text{cg}}$

$\therefore x = 15$ hg

Clave C

5. 1 hm _____ 1000 dm
0,2 hm _____ x

$\Rightarrow x = \dfrac{0{,}2\ \text{hm}.1000\ \text{dm}}{1\ \text{hm}}$

$\therefore x = 200$ dm

Clave D

6. 1 dam _____ 1000 cm
x _____ 2×10^5 cm

$\Rightarrow x = \dfrac{2 \times 10^5\ \text{cm}.1\ \text{dam}}{1000\ \text{cm}} = 200$ dam

Clave C

7. 8×10^6 ml _____ x
10^6 ml _____ 1 kl

$\Rightarrow x = \dfrac{8 \times 10^6\ \text{ml}.1\ \text{kl}}{10^6\ \text{ml}} = 8$ kl

Clave B

8. $1\text{ h} \;\_\_\_\_\_\_\; 10^4\text{ cl}$

$0{,}05\text{ hl} \;\_\_\_\_\_\_\; x$

$\Rightarrow x = \dfrac{0{,}05\text{hl} \times 10^4\text{cl}}{1\text{hl}} = 500\text{ cl}$

Clave A

Resolución de problemas

9. Adulto $\Rightarrow$ 75 kg

Niño $\Rightarrow$ 3,5 mag = 35 kg

$75x + 35y = 400$ kg

1	9
2	7
3	5
4	2
5	0

$x = 1 \wedge y = 9 \Rightarrow \underbrace{x + y}_{\text{máx.}} = 10$

Clave B

10. $d = (28\text{ hm} + 0{,}75\text{ km} + 250\text{ dam}) \times 2$

$d = (2{,}8\text{ km} + 0{,}75\text{ km} + 2{,}5\text{ km}) \times 2$

$d = (6{,}05\text{ km}) \times 2 = 12{,}1\text{ km}$

Clave C

11. Capacidad llenada por día: Cd

Cd = A + B + C

Cd = 7,5 kl + 50 l + 35 dal

Cd = 75 hl + 0,5 hl + 3,5 hl

Cd = 79 hl

Hallamos el n.° de días:

$\text{N.° días} = \dfrac{1185\text{hl}}{79\text{hl}} = 15$

Clave C

Nivel 2 (página 77) Unidad 4

Comunicación matemática

12.

13.

Razonamiento y demostración

14.
- 0,2 mag = 0,2 (100 hg) = 20 hg
- 50 dag = 50(0,1 hg) = 5 hg
- 20 hg + 5 hg = 25 hg

Clave B

15. $x\text{ dm} + 0{,}02\text{ m} = 40\text{ mm}$

$x\text{ dm} + 0{,}02\ (10\text{ dm}) = 40\ (10^{-2}\text{ dm})$

$x = 0{,}4 - 0{,}2 = 0{,}2$

Clave D

16. $x\text{ hl} + 40\text{ dal} = 2\text{ kl}$

$x(0{,}1\text{ kl}) + 40(10^{-2}\text{ kl}) = 2\text{ kl}$

$(0{,}1)x + 0{,}4 = 2$

$0{,}1x = 1{,}6 \Rightarrow x = 16$

Clave B

17. $0{,}2\text{ g} - x\text{ cg} = 150\text{ mg}$

$200\text{ mg} - (10\text{ mg})x = 150\text{ mg}$

$50 = 10x \Rightarrow x = 5$

Clave C

18. $8\text{ hm} - x\text{m} = 0{,}035\text{ km}$

$800\text{ m} - x\text{m} = 35\text{ m}$

$x = 765$

Clave E

19. $450\text{ dal} + 350\text{ l} = x\text{ kl}$

$450(10^{-2}\text{ kl}) + 350(10^{-3}\text{ kl}) = x\text{ kl}$

$4{,}5\text{ kl} + 0{,}35\text{ kl} = x\text{ kl}$

$x = 4{,}85$

Clave A

Resolución de problemas

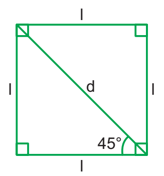
20.
- $1{,}7\text{ t} \times 7 = 11{,}9\text{ t}$
- $15\text{ q} \times 12 = 1{,}5\text{ t} \times 12 = 18\text{ t}$
- $9 \times 10^3\text{ hg} \cdot x = 0{,}9\text{ t}\ x = 0{,}9 \times \text{t}$
- $11{,}9\text{ t} + 18\text{ t} + 0{,}9x\text{ t} = 47\text{ t}$

  $29{,}9 + 0{,}9x = 47$

  $x = 19$

Clave B

21.



$\Rightarrow d = \sqrt{2}\,l$

- $d = \sqrt{2}\text{ dm} = \sqrt{2}\ l \Rightarrow l = 1\text{ dm}$
- Perímetro = p = 4 l

  p = 4(1 dm) = 4 dm

  p = 4(10 cm) $\therefore$ p = 40 cm

Clave A

22.

N.° botellas	Capacidad
3x	$50\text{ cl} = 50 \times 10^{-3}\text{ dal}$
x	$2{,}5\text{ dl} = 2{,}5 \times 10^{-2}\text{ dal}$
Total de botellas	35 dal
4x	

Luego:

$3x(50 \times 10^{-3}\text{ dal}) + x(2{,}5 \times 10^{-2}\text{ dal}) = 35\text{ dal}$

$x(0{,}15 + 0{,}025) = 35$

$x(0{,}175) = 35$

$x = 200$

$\therefore 4x = 800$

Clave C

Nivel 3 (página 78) Unidad 4

Comunicación matemática

23.
- M = 450 mg + 25 cg + 0,01 g + 0,28 dag

  M = 3285 mg
- N = 0,02 hg + 46 dg − 0,5 dag + 5g

  N = 6600 mg
- P = 0,004 kg + 300 mg + 40 cg − 20 dg

  P = 2700 mg

$\therefore$ N > M > P

Clave D

24.

Razonamiento y demostración

25. $x\text{ cg} + 0{,}1\text{ dag} = 45\text{ g}$

$x\text{ cg} + 0{,}1 \times 10^3\text{ cg} = 45 \times 10^2\text{ cg}$

$x + 100 = 4500$

$\therefore x = 4400$

Clave B

26. $x\text{ cg} + 32\text{ dg} = 0{,}08\text{ hg} + 0{,}004\text{ dag}$

$\therefore x = 484\text{ cg}$

Clave D

27. $k\text{ dal} + 42\text{ l} = 925\text{ dl} + 300\text{ cl}$

$k = 9{,}25\text{ dal} + 0{,}3\text{ dal} - 4{,}2\text{ dal}$

$k = 5{,}35\text{ dal}$

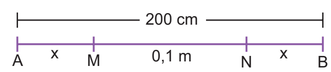
Clave A

28.

	kl	hl	dal	l	dl	cl	ml
A)		0,	0	3			
B)			0,	3			
C)				3	0	0	
D)			3	0	0	0	
E)				3	0		

Clave D

29.



$\Rightarrow 200\text{ cm} = 2x + 0{,}1\text{ m}$

$200\text{ cm} = 2x + 100\text{ cm}$

$100\text{ cm} = 2x \Rightarrow x = 50\text{ cm}$

$\therefore x = 5\text{ dm}$

Clave C

30. $p = \overline{AB} + \overline{BC} + \overline{AC}$

$0{,}0073\text{ km} = k + 0{,}02\text{ hm} + 350\text{ cm}$

$7{,}3\text{ m} = k + 2\text{ m} + 3{,}5\text{ m}$

$\Rightarrow k = 1{,}8\text{ m}$

Clave D

31. De la figura:

AB = 32m − x ... (1)

Dado: BD = 0,46 km = 460 m

$\Rightarrow$ CD − 460m − x ... (2)

También AB + CD = 2520 dm = 252 m ... (3)

(1) y (2) en (3):

$\Rightarrow 32\text{ m} - x + 460\text{ m} - x = 252\text{ m}$

$240\text{ m} = 2x$

$x = 120\text{ m} = 12\text{ dam}$

Clave B

Resolución de problemas

32. Como nos piden en cm^2:

$\Rightarrow 7 \cdot 10^{-5}\text{ hm a cm}$

$1\text{ hm} \;\_\_\_\_\_\_\; 10^4\text{ cm}$

$7 \cdot 10^{-5}\text{ hm} \;\_\_\_\_\_\_\; x$

$x = 0{,}7\text{ cm}$

$\therefore A_{\square} = x^2 = 0{,}49\text{ cm}^2$

Clave B

33. Pasamos a m (metros):

$$M = \frac{10m}{70m} + \frac{60m}{70m}$$

Simplificamos:

$$M = \frac{1}{7} + \frac{6}{7} + \frac{7}{7} = 1$$

Clave E

34. En x días:

(50 hl)(x) – (0,40 kl)x = 9000 dl

500 l(x) – 400 l(x) = 900 l

100x = 900l

$\therefore$ x = 9 días

Clave E

35. Hallamos la cantidad de tela necesaria para vestir a 1 persona: C_t

C_t = 32 dm + 0,02 hm + 80 cm + 1500 mm

C_t = 5,2 m + 2 m + 0,8 m + 1,5 m

C_t = 9,5 m

Para 10 personas (x):

x = (9,5 m)10 = 95 m

Clave B

36. Todos a metros:

n.° 17: 500 m → 2.° lugar

n.° 14: 300 m → 3.° lugar

n.° 21: 1000 m

n.° 5: 180 m → 1.°

Clave C

37. La ecuación es:

$$x \text{ dam} + x \text{ dam} + 2{,}8 \text{ km} = 11 \text{ km}$$
$$2x \,.\, 10^{-2} \text{ km} = 8{,}2 \text{ km}$$
$$x = 4{,}1 \,.\, 10^{2} \text{ km}$$
$$x = 41 \,.\, 10^{4} \text{ m}$$
$$x = 41 \,.\, 10^{5} \text{ dm}$$

Clave D

MARATÓN MATEMÁTICA (página 80)

1. Por ángulos cuadrantales sabemos:

$$M = \frac{0-(-1)+0}{1-(-1)+1} = \frac{1}{3}$$

$$\therefore M = \frac{1}{3}$$

Clave D

2. P es punto medio:

$$\Rightarrow P(x; y) = \frac{A+B}{2}$$

$$x = \frac{0+(-4)}{2} = -\frac{4}{2} = -2$$

$$y = \frac{3+(-3)}{2} = 0$$

$\therefore$ P(x; y) = (–2; 0)

Clave E

3.

$$\cot\theta = \frac{-4}{-3} = \frac{m}{-6} \Rightarrow m = -8$$

Clave B

4.

- $0° < \alpha < 90° \ \wedge \ 180° < \beta < 270°$

  $90° < \beta - \alpha < 270°$

  $\Rightarrow \text{sen}(\beta - \alpha) = (-) \vee (+)$

- $0 < 2\alpha < 180°$

  $\text{sen}2\alpha = (+)$

  $\therefore (+) \vee (-) ; (+)$

Clave B

5. Tenemos:

$$M = \frac{\tan 1125°}{\sqrt{2}\csc 405°} = \frac{\tan(360° \times 3 + 45°)}{\sqrt{2}\csc(360° + 45°)}$$

$$M = \frac{\tan 45°}{\sqrt{2}\csc 45°} = \frac{1}{\sqrt{2}(\sqrt{2})}$$

$$\therefore M = \frac{1}{2}$$

Clave A

6. sen91° = sen(90° + 1°) = cos1°

sen92° = sen(90° + 2°) = cos2°

⋮ ⋮ ⋮

sen125° = sen(90° + 35°) = cos35°

Reemplazamos:

$$M = \frac{\cos 1° + \cos 2° + ... + \cos 35°}{\cos 1° + \cos 2° + ... + \cos 35°}$$

$\therefore$ M = 1

Clave D

7. Reducimos:

$$M = \text{sen}(\pi - \theta) + \cos\left(\theta - \frac{3\pi}{2}\right)$$

$$M = \text{sen}\theta + \cos\left[-\left(\frac{3\pi}{2} - \theta\right)\right]$$

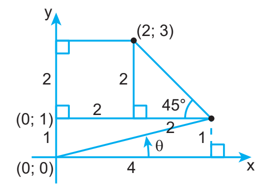
$$M = \text{sen}\theta + \cos\left(\frac{3\pi}{2} - \theta\right)$$

$$M = \text{sen}\theta - \text{sen}\theta = 0$$

$\therefore$ M = 0

Clave C

8. Tenemos:



Entonces:

$$\therefore \tan\theta = \frac{1}{4}$$

Clave D

9. $L_1 = 3x - 4y - 3 = 0$

Sabemos: $(y - y_0) = m(x - x_0)$

Entonces:

$3x - 3 = 4y$

$3(x - 1) = 4y \Rightarrow y = \frac{3}{4}(x - 1)$

$y - 0 = \frac{3}{4}(x - 1)$

$\therefore m = \frac{3}{4}$

Clave B

10.

$\sqrt{\tan\theta} > 0 \Rightarrow \tan\theta > 0$

$\wedge \ \text{sen}\theta < 0 \quad \Rightarrow \theta \in \text{IIIC}$

I. $\cos\theta < 0$

II. $\dfrac{\tan 100°}{\text{sen}\theta} = \dfrac{(-)}{(-)} = (+)$

$\therefore (-) ; (+)$

Clave A